

Please write clearly in block capitals.

Centre number

--	--	--	--	--

Candidate number

--	--	--	--

Surname

WRITTEN SOLUTIONS

Forename(s)

Candidate signature

GCSE MATHEMATICS

H

Higher Tier

Paper 1 Non-Calculator

Date of Exam

Morning

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- mathematical instruments.

You must **not** use a calculator.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

Answer all questions in the spaces provided.

- 1 What is the area, in
- cm^2
- , of a semicircle of radius 6 cm?



Circle your answer.

[1 mark]

6π

12π

18π

36π

$$\text{AREA OF CIRCLE} = \pi r^2 = \pi \times 6^2 = \pi \times 6 \times 6 = 36\pi$$

$$\text{BUT FOR SEMI CIRCLE WE ONLY NEEDED HALF OF THIS: } 36\pi \div 2 = 18\pi$$

- 2 Expand

$$3x^2(2x-5)$$

$$3x^2 \times 2x = 3 \times 2 \times x^2 \times x = 6x^3$$

Circle your answer.

[1 mark]

$$3x^2 \times (-5)$$

$-9x$

$6x^3 - 5$

$5x^3 - 8x^2$

$6x^3 - 15x^2$

$$= 3 \times (-5) \times x^2$$

$$= -15x^2 \quad \text{SO } 6x^3 - 15x^2$$

- 3 Circle the solution of
- $2x + 8 > 4$
- (REPLACE
- $>$
- WITH
- $=$
-)

[1 mark]

$x > -6$

$x > -2$

$x > 2$

$x > 6$

$$\text{Let } 2x + 8 = 4 \quad \text{AND SOLVE}$$

$$\quad \quad -8 \quad \quad -8$$

$$2x = -4$$

$$\div 2 \quad \quad \div 2$$

$$x = -2$$

NOW PUT THE INEQUALITY SIGN BACK

$$x > -2$$

- 4 Circle the calculation that increases 50 by 200%

[1 mark]

50×1.2

50×2

50×2.2

50×3

50 = 100%. so 200% is $50 \times 2 = 100$
 TO INCREASE 50 BY 200%. ADD 100 TO 50 = 150
 so 50×3

5

Solve $\frac{x}{3} - 9 = 12$

$+9 \quad +9$

[2 marks]

$\times 3 \quad \frac{x}{3} = 21 \quad \times 3$

$x = 63$

$x = 63$

Turn over for the next question

Turn over ►

- 6 The air pressure in a tyre measures 7.2 bar.
Air is leaking out at the rate of 0.2 bar per day.

- 6 (a) Assume that the air continues to leak at the same rate.

After how many days will the pressure measure 4.8 bar?

[2 marks]

$$\begin{array}{r} 7.2 \\ - 4.8 \\ \hline 2.4 \end{array} \quad 2.4 \div 0.2 = 24 \div 2 = 12$$

so 12 days

Answer

12

- 6 (b) In fact, the rate that the air leaks out increases each day.

How does this affect your answer to part (a)?

[1 mark]

'REDUCE THE NUMBER OF DAYS TAKEN'

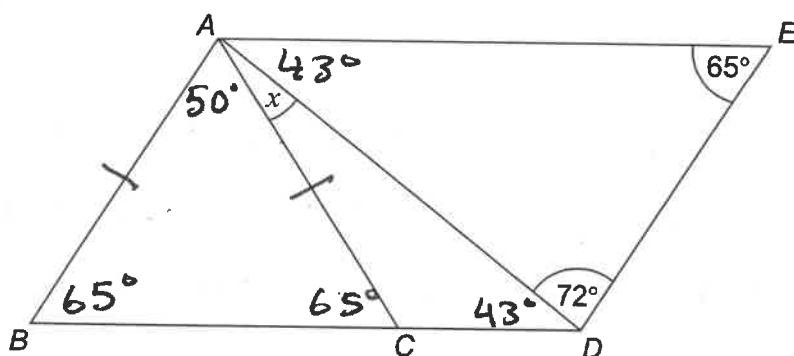
OR

'IT WILL BE QUICKER'

(OR SIMILAR ANSWER)

7 $ABDE$ is a parallelogram.

$AB = AC$



Not drawn
accurately

$$\begin{array}{r} 72 \\ + 65 \\ \hline 137 \\ \overset{7}{1}80 \\ - 137 \\ \hline 43 \end{array}$$

Show that $x = 22^\circ$

[3 marks]

$$\angle DAE = 180 - 72 - 65 = 43^\circ$$

$$\angle CBA = 65^\circ \text{ (OPPOSITE ANGLES EQUAL)}$$

$$\angle ADC = 43^\circ \text{ (ALTERNATE ANGLES IN PARALLEL LINES)}$$

AS $AB = AC$, TRIANGLE ABC IS ISOSCELES SO

$$\angle ACB = 65^\circ$$

$$\angle BAC = 180 - 65 - 65 = 180 - 130 = 50^\circ$$

$$\angle BAC = \angle BDE \text{ (OPPOSITE ANGLES IN PARALLELOGRAM)}$$

$$\text{SO } x + 50 + 43 = 72 + 43$$

$$x + 93 = 115$$

$$x = 115 - 93$$

$$x = 22^\circ$$

Turn over ►

- 8 (a) Here are the fourth and fifth terms of a Fibonacci-type sequence.

$$z \quad y=13 \quad x=15 \quad 28 \quad 43$$

$$43-28$$

Each term is the sum of the previous two terms.

Show that the first term is 2

$$\begin{array}{r} 3 \quad 1 \quad 28 \\ \cancel{4} \quad 3 \\ \hline 28 \\ \hline 15 \end{array} \quad \begin{array}{r} 28 \\ -15 \\ \hline 13 \end{array}$$

[2 marks]

$$x + 28 = 43, \text{ so } x = 43 - 28 = 15$$

$$y + 15 = 28, \text{ so } y = 28 - 15 = 13$$

$$z + 13 = 15 \text{ so } z = 15 - 13 = 2$$

$$z = 2$$

- 8 (b) Here are the first and third terms of a different Fibonacci-type sequence.

$$a \quad b-a \quad b \quad 2b-a$$

Each term is the sum of the previous two terms.

Work out an expression in terms of a and b for the fifth term.

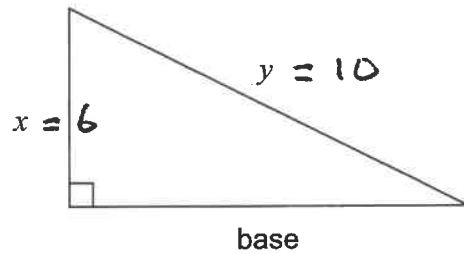
[3 marks]

$$\begin{aligned} \text{2nd Term} & \text{ must be } b-a \\ \text{4th Term} & = b + (b-a) = 2b-a \\ \text{5th Term} & = (2b-a) + b = 3b-a \end{aligned}$$

Answer

$$3b - a$$

- 9 Noah is attempting to work out the base of **different** right-angled triangles.



Not drawn
accurately

From Pythagoras

$$\begin{aligned} \text{BASE}^2 &= y^2 - x^2 \\ &= 10^2 - 6^2 \\ &= 100 - 36 \\ &= 64 \end{aligned}$$

Here is his method with the working for $y = 10$ and $x = 6$

Work out the value of y^2

$$10^2 = 100$$

$$\begin{aligned} \text{BASE} &= \sqrt{64} \\ &= 8 \end{aligned}$$

Work out the value of x^2

$$6^2 = 36$$

Work out the value of $y^2 - x^2$

$$100 - 36 = 64$$

The base is $\sqrt{y^2 - x^2}$

$$\text{base} = \sqrt{64}$$

Tick the correct statement.

[3 marks]

The method will **always** give an answer which is a whole number.

The method will **sometimes** give an answer which is a whole number.

The method will **never** give an answer which is a whole number.

Show working to support your answer.

From above,

$$\sqrt{64} = 8 = \text{WHOLE NUMBER}$$

But try another example

$$\begin{aligned} \text{BASE} &= \sqrt{12^2 - 6^2} = \sqrt{144 - 36} = \sqrt{108} \\ &= \sqrt{85} \end{aligned}$$

$\sqrt{85}$ IS NOT A WHOLE NUMBER

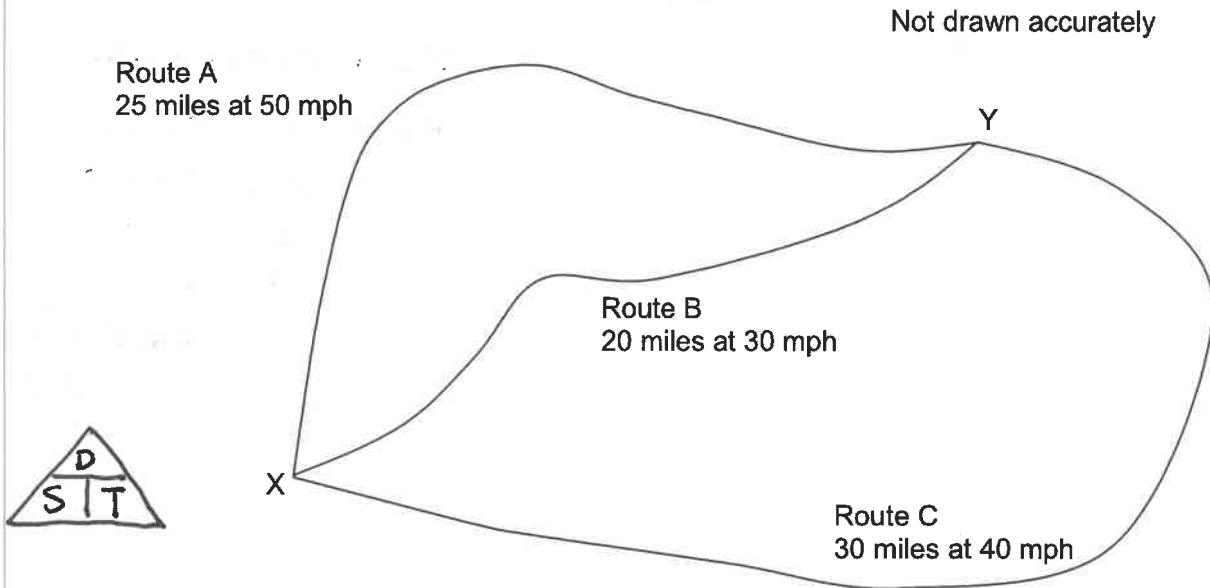
i.e. $9^2 = 81$, $10^2 = 100$, so answer is a decimal



$$\begin{array}{r} 121 \\ - 36 \\ \hline 85 \end{array}$$

Turn over ►

- 10 The diagram shows three routes, A, B and C, between two towns, X and Y.
The distance and average speed for each route is shown.



- 10 (a) Which of the three routes takes the longest time?

Assume the average speeds given.

You **must** show your working.

[4 marks]

$$T = \frac{D}{S} \quad \text{For A: } T = \frac{25}{50} = 0.5 \text{ HOURS}$$

$$\text{For B: } T = \frac{20}{30} = \frac{2}{3} = 0.6 \text{ HOURS}$$

$$\text{For C: } T = \frac{30}{40} = 0.75 \text{ HOURS}$$

Answer C = 0.75 hours = 45 mins.

10 (b) Jon and Matt take the same time to travel from X to Y.

Jon travels along route B at 10 mph **faster** than the average speed.

Matt travels along route C.

Does Matt travel faster or slower than the average speed for route C, and by how much?

You **must** show your working.

[3 marks]

Jon: Route B, 20 miles at 40 mph

$$T = \frac{20}{40} = 0.5 \text{ hours} = 30 \text{ mins}$$

Matt travels route C in same time = 30 mins
= 0.5 hours

Average = 45 mins

$$\text{so } 45 - 30 = 15 \text{ mins}$$

$$\text{so speed} = \frac{D}{T} = 30 \div 0.5 = 60 \text{ mph}$$

Tick a box.

Faster

Slower

Answer

15 mins

mph

NEW SPEED = 60 mph ~~which is twice as fast~~

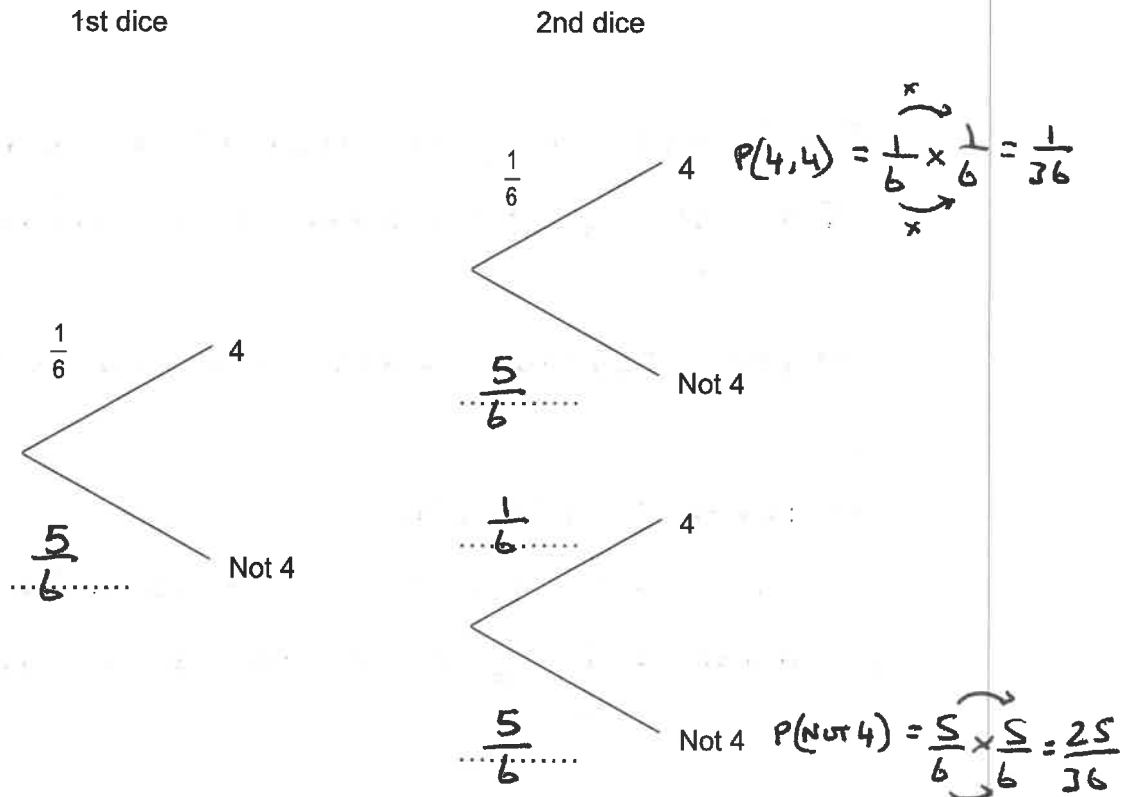
Turn over for the next question

Turn over ►

11 Two ordinary fair dice are rolled.

11 (a) Complete the tree diagram.

[1 mark]



11 (b) Circle the probability that **both** dice land on 4

[1 mark]

$$\frac{1}{4}$$

$$\frac{2}{12}$$

$$\frac{2}{6}$$

$$\frac{1}{12}$$

$$\frac{1}{36}$$

11 (c) Work out the probability that at least one of the dice does **not** land on 4

[2 marks]

$$\begin{aligned}
 &P(\text{NOT } 4) \text{ FOR AT LEAST ONE DICE} \\
 &= 1 - P(\text{BOTH DICE ON } 4) \\
 &= 1 - \frac{1}{36} = \frac{35}{36}
 \end{aligned}$$

Answer

$$12 \quad A = \frac{(x-4)(x+3)}{x(x-1)} = \frac{(-1-4)(-1+3)}{-1(-1-1)}$$

12 (a) Work out the value of A when $x = -1$

[1 mark]

$$= \frac{(-5) \times (2)}{-1 \times (-2)} = \frac{-10}{2} = -5$$

Answer

12 (b) When $2 < x < 4$ Let $x = 3$

Circle your answer.

[1 mark]

A is positive

A is zero

A is negative

A could be positive or negative or zero

$$\frac{(3-4)(3+3)}{3(3-1)} = \frac{(-1) \times 6}{3 \times 2} = \frac{-6}{6} = -1$$

Turn over for the next question

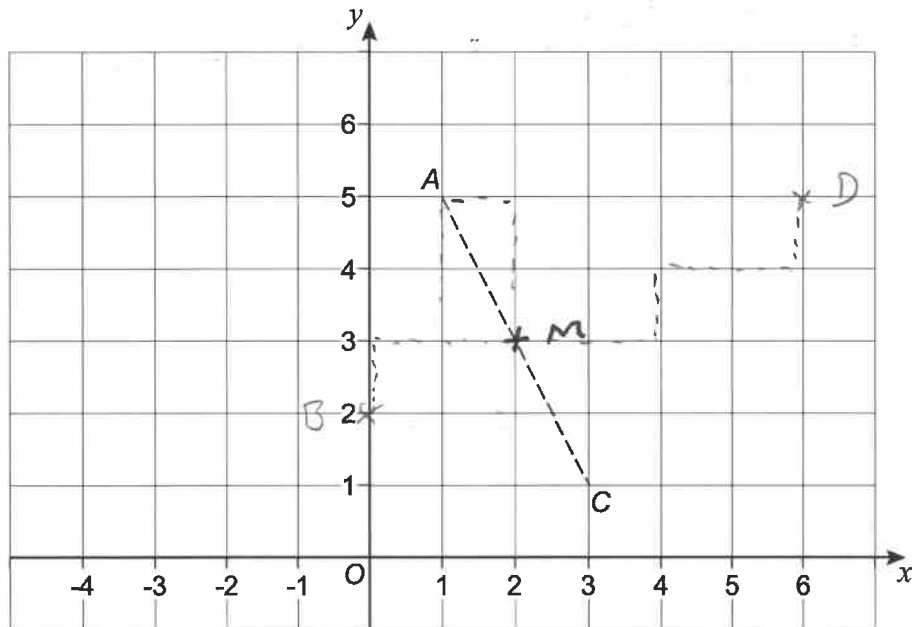
AS x IS NOT BIGGER THAN 4, THE FIRST BRACKET WILL ALWAYS BE NEGATIVE.

Turn over ►

13 (a) AC is a diagonal of kite $ABCD$.

A is the point $(1, 5)$

C is the point $(3, 1)$



The diagonals of the kite intersect at M , the midpoint of AC .

$$AM = BM$$

$$BM : MD = 1 : 2$$

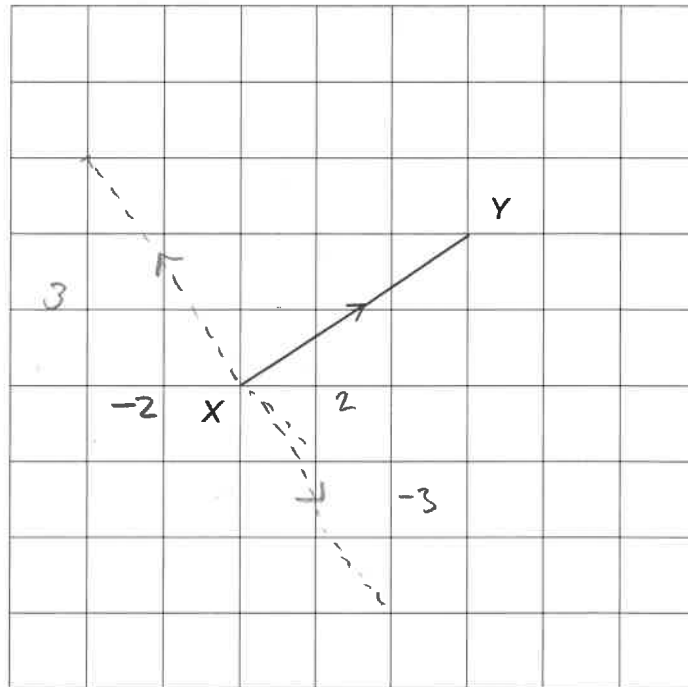
Work out possible coordinates of B and D .

[2 marks]

$$B(0, 2) \text{ and } D(6, 5)$$

$$\text{or } B(4, 4) \text{ and } D(-2, 1)$$

13 (b) \vec{XY} is the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ on this square grid.



Write down a vector that is

the same size as \vec{XY}

and perpendicular to \vec{XY}

[2 marks]

Answer $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Turn over for the next question

14 Estimate the value of $19.4^2 + 30\sqrt{104}$

[3 marks]

$$\begin{aligned} &\approx 20^2 + 30 \times \sqrt{100} \\ &= 400 + 30 \times 10 \\ &= 400 + 300 \\ &= 700 \end{aligned}$$

Answer 700

15

Circle the expression that is equivalent to $\frac{2x^2+1}{x}$ where x is not equal to 0

[1 mark]

$2x + 1$

$2x^2 + \frac{1}{2}$

$2x + \frac{1}{x}$

$4x + \frac{1}{x}$

$$\frac{2x^2+1}{x} = \frac{2x^2}{x} + \frac{1}{x}$$

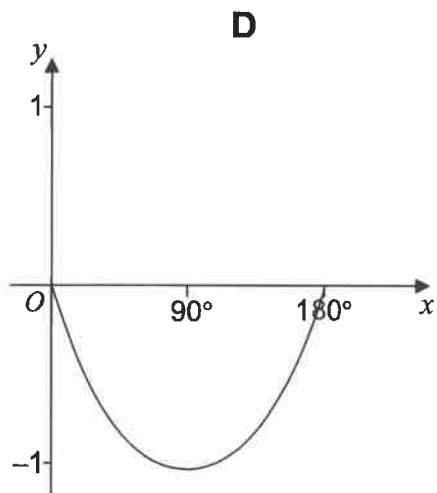
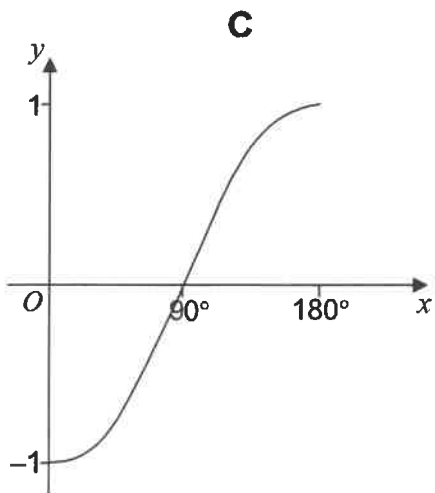
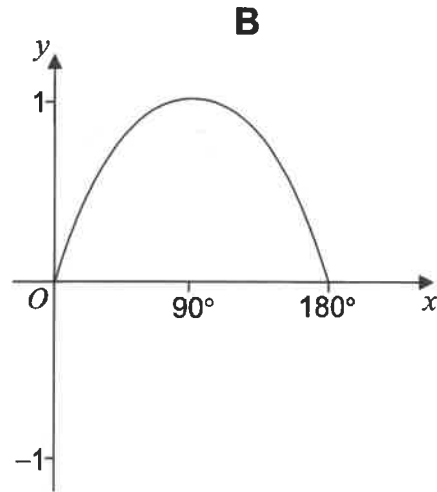
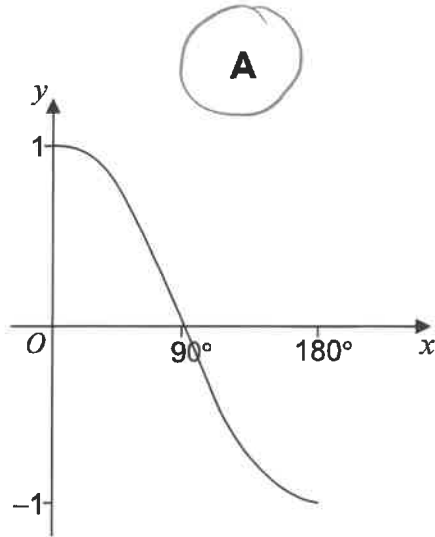
$$= 2x + \frac{1}{x}$$

16 One of these is a sketch of $y = \cos x$ for $0^\circ \leq x \leq 180^\circ$

Which one?

Circle the correct letter.

[1 mark]



Turn over for the next question

Turn over ►

17

Naz buys a fridge from a shop for £189

The cost of delivery is proportional to the distance from the shop.

For 15 miles, the cost is £9

Naz lives 24 miles from the shop.

Is the total cost **more** than £200?You **must** show your working.

[4 marks]

(Delivery) Cost \propto DIST.

$$C \propto D \quad \text{so} \quad C = kD \quad \text{FIND } k$$

$$C = \pounds 9 \quad \text{when} \quad D = 15$$

$$\text{so} \quad 9 = k \times 15$$

$$\frac{3}{5} = \frac{9}{15} = k$$

$$\underline{\underline{\text{so}}} \quad C = \frac{3}{5} \times D$$

$$\text{Cost} = \frac{3}{5} \times 24 \text{ miles}$$

$$\text{Cost} = \frac{24 \times 3}{5} = \frac{72}{5} = 14.40$$

$$\text{TOTAL COST} = \pounds 189 + 14.40 = 203.40$$

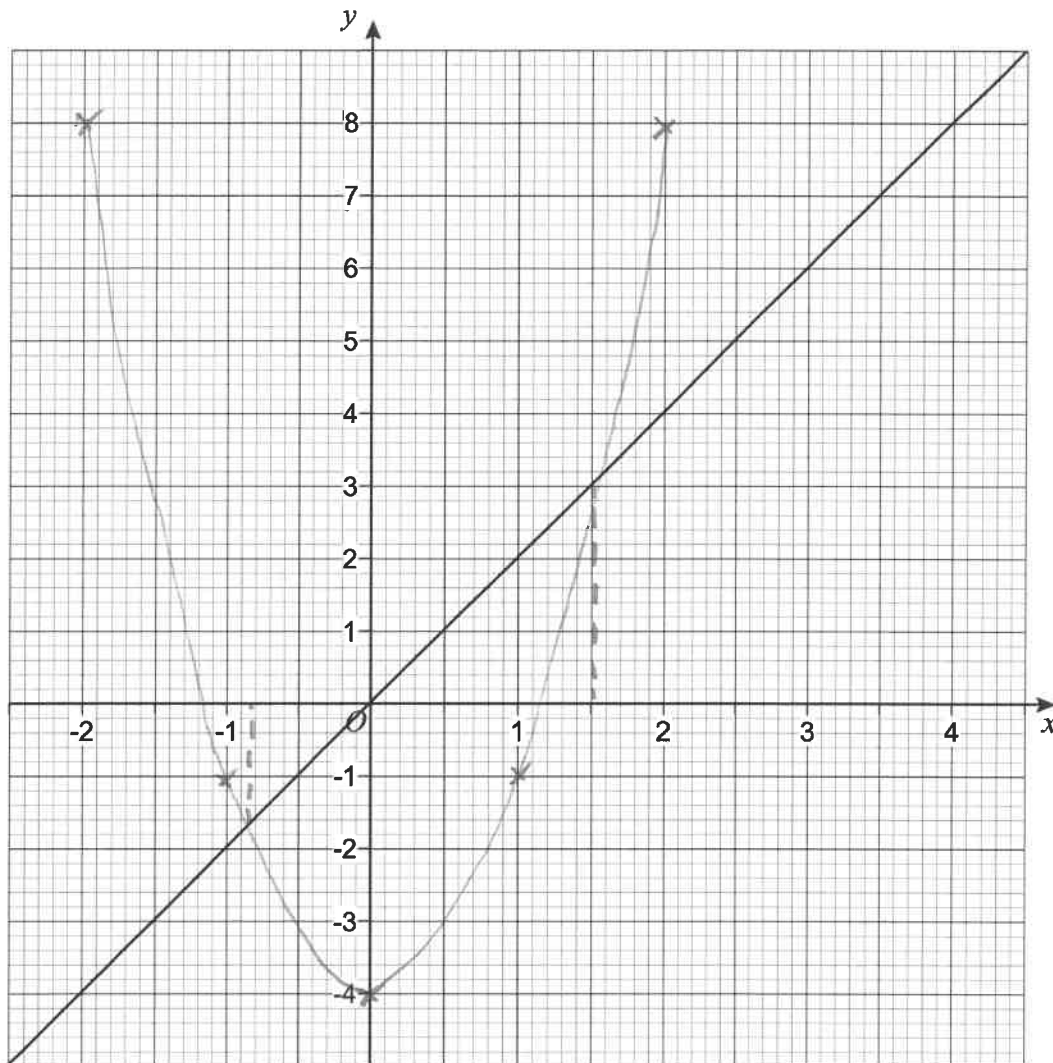
Answer YES, 203.40

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \end{array}$$

$$\begin{array}{r} 14.40 \\ 5 \overline{) 72.00} \end{array}$$

$$\begin{array}{r} 189.00 \\ 14.40 \\ \hline 203.40 \end{array}$$

18 The graph of $y = 2x$ is shown.



By drawing the graph $y = 3x^2 - 4$ on the grid,
work out approximate solutions to $3x^2 - 4 = 2x$

[4 marks]

Let $x =$ -2 -1 0 1 2 3 4
 $x^2 =$ 4 1 0 1 4 9 16
 $3x^2 =$ 12 3 0 3 12 27
 $-4 =$ 8 -1 -4 -1 8 23
 PLOT $(-2, 8)$ $(-1, -1)$ $(0, -4)$ $(1, -1)$ $(2, 8)$

Answer $x = -0.85, 1.5$

Approximately

Turn over ►

19 (a) Work out the value of $(\sqrt{2})^4$ [1 mark]

$$= (2^{\frac{1}{2}})^4 = 2^{(\frac{1}{2} \times 4)} = 2^2 = 4$$

Answer 4

19 (b) Expand and simplify $(\sqrt{2} + 3)^2$ [2 marks]

$$\begin{aligned}
 & (\sqrt{2} + 3)(\sqrt{2} + 3) && (\sqrt{2} \times \sqrt{2} = 2) \\
 & = \sqrt{2} \times \sqrt{2} + 3\sqrt{2} + 3\sqrt{2} + 3 \times 3 \\
 & = 2 + 6\sqrt{2} + 9 \\
 & = 11 + 6\sqrt{2}
 \end{aligned}$$

Answer $11 + 6\sqrt{2}$

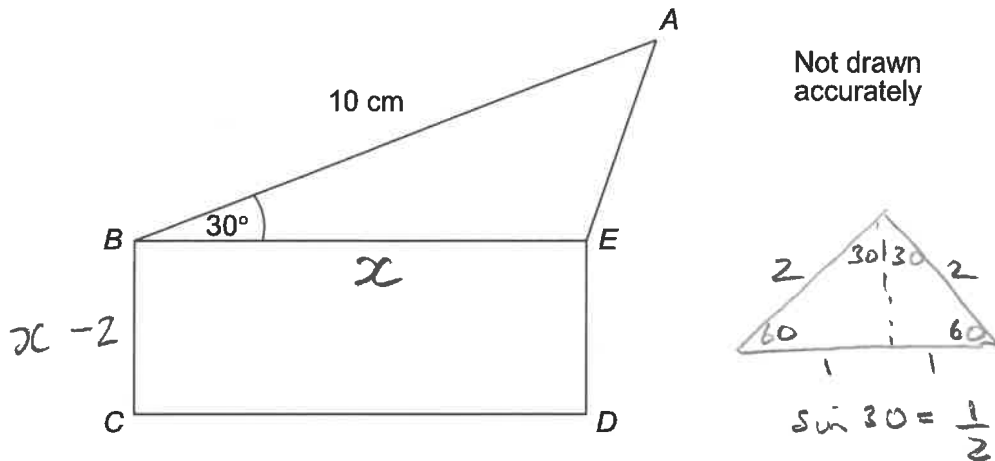
20 Work out the value of $9^{-\frac{1}{2}}$ [2 marks]

$$\begin{aligned}
 & = \frac{1}{9^{\frac{1}{2}}} \\
 & = \frac{1}{\sqrt{9}} \\
 & = \frac{1}{3}
 \end{aligned}$$

Answer $\frac{1}{3}$

(OR 0.33)

21

The diagram shows a triangle ABE and a rectangle $BCDE$.area $ABE = \text{area } BCDE$ BC is 2 cm shorter than BE .Work out the length of BE .

[4 marks]

$$\text{AREA OF } BCDE = x(x-2) = x^2 - 2x$$

$$\text{AREA OF TRIANGLE} = \frac{1}{2} \times x \times 10 \times \sin 30$$

$$= \frac{1}{2} \times 10 \times \frac{1}{2} \times x$$

$$= 2.5x$$

$$\text{so } x^2 - 2x = 2.5x$$

$$x^2 - 4.5x = 0$$

$$x(x - 4.5) = 0$$

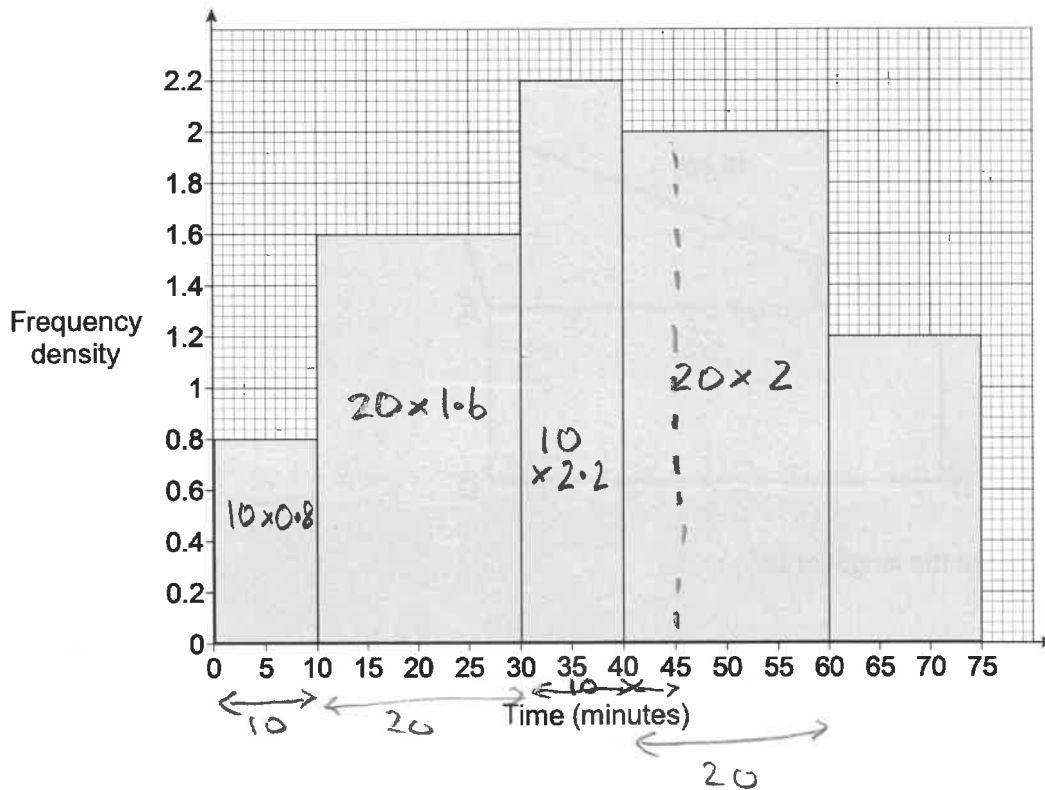
~~$x = 0$~~ $x = 4.5 \text{ cm } \checkmark$

Answer 4.5 cm cm

Turn over ►

22

The histogram shows information about the times some students revised for a test.
The first bar represents students who revised for less than 10 minutes.



Estimate the number of students who revised for less than 45 minutes.

[3 marks]

$$10 \times 0.8 = 8$$

$$20 \times 1.6 = 2 \times 16 = 32$$

$$10 \times 2.2 = 22$$

$$20 \times 2 = 40 \text{ (ONLY NEED ONE QUARTER OF THIS = 10)}$$

Answer

72

$$8 + 32 + 22 + 10 = 72$$

23

Work out the value of $\frac{5}{\sqrt{3}} - \sqrt{\frac{3}{4}}$

$$6 \frac{+3}{4} = \frac{24+3}{4} = \sqrt{\frac{27}{4}}$$

Give your answer in the form $k\sqrt{3}$

[4 marks]

$$\sqrt{\frac{27}{4}} = \frac{\sqrt{27}}{\sqrt{4}} = \frac{\sqrt{9 \times 3}}{2} = \frac{\sqrt{9} \times \sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$$

 $\frac{5}{\sqrt{3}}$ RATIONALISE THE DENOMINATOR

$$= \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

$$\frac{5\sqrt{3}}{3} - \frac{3\sqrt{3}}{2} = \frac{10\sqrt{3} - 9\sqrt{3}}{6} = \frac{\sqrt{3}}{6}$$

Answer

$$\frac{1}{6} \sqrt{3}$$

24

Convert $0.\dot{2}8$ to a fraction.

Give your answer in its simplest form.

[3 marks]

$$\text{Let } x = 0.2888 \dots$$

$$10x = 2.8888 \dots$$

$$100x = 28.8888$$

$$- 10x = -2.8888$$

$$90x = 26$$

$$x = \frac{26}{90} = \frac{13}{45}$$

Answer

$$\frac{13}{45}$$

Turn over ►

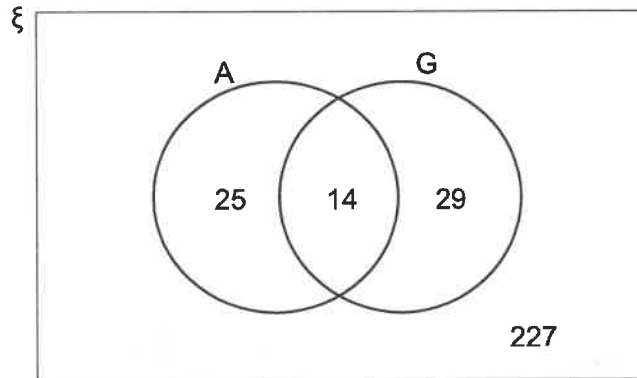
25

In the Venn diagram

 ξ = 295 students in a college

A = students who take Art

G = students who take Geography



25 (a) One student is chosen at random.

Work out the probability the student takes Art.

[1 mark]

39 ART STUDENTS OUT OF 295

Answer

$$\frac{39}{295}$$

25 (b) One student who takes Geography is chosen at random.

Work out the probability the student **also** takes Art.

[1 mark]

TOTAL GEOGRAPHY STUDENTS = 14 + 29 = 43

Answer

$$\frac{14}{43}$$

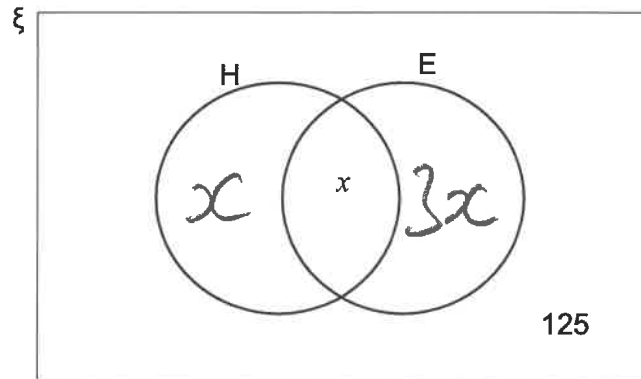
$$\text{so } \frac{14(G+A)}{43}$$

25 (c) In this Venn diagram

ξ = 295 students in the college

H = students who take History

E = students who take English



One-half of the students who take History also take English. SO HISTORY ONLY = $2x$
The number who take English is twice the number who take History.

Work out the value of x .

[3 marks]

$$\text{TOTAL HISTORY} = 2x$$

$$\text{TOTAL ENGLISH} = 2x \times 2 = 4x \quad (\text{BUT } x \text{ ALREADY IN DIAGRAM})$$

$$\text{SO } 5x + 125 = 295$$

$$\text{Answer } x = 34$$

$$5x = 295 - 125$$

$$5x = 170$$

$$x = 34$$

Turn over for the next question

Turn over ►

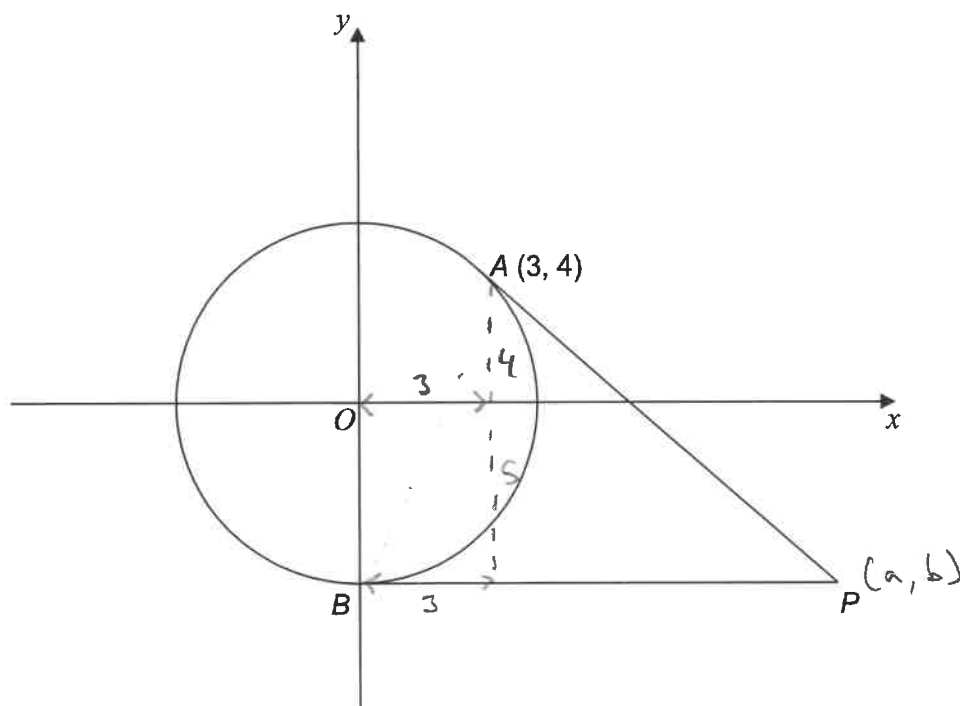
26

A and B are points on the circle with equation $x^2 + y^2 = 25$

A is $(3, 4)$

B is a point on the y -axis.

PA and PB are tangents.



26 (a) Show that the coordinates of B are $(0, -5)$

[1 mark]

$$x^2 + y^2 = 25 = r^2$$

$$\text{RADIUS} = \sqrt{25} = 5, \text{ so } OB = 5 \text{ UNITS}$$

$$\text{so } B = (0, -5)$$

26 (b) Give a reason why $PA = PB$

[1 mark]

TANGENTS FROM A COMMON POINT
OUTSIDE A CIRCLE ARE ALWAYS EQUAL
IN LENGTH.

26 (c) P is the point (a, b)

Work out the values of a and b.

[4 marks]

$$\text{For } P, b = -5$$

For a: METHOD 1 - CONSIDER GRADIENTS

$$\text{GRADIENT OF } OA = \frac{4}{3}$$

$$\text{So GRADIENT OF } AP = -\frac{3}{4} \text{ (PERPENDICULAR TO } OA)$$

$$\text{GRADIENT} = \frac{\text{CHANGE IN } Y}{\text{CHANGE IN } X} \quad A = (3, 4) \quad P = (a, -5)$$

$$\text{GRADIENT } AP = \frac{-5-4}{a-3} = \frac{-9}{a-3} \text{ BUT THIS EQUALS } -\frac{3}{4}$$

$$a = 15$$

$$b = -5$$

$$\frac{-9}{a-3} = -\frac{3}{4} \text{ (CROSS MULTIPLY)}$$

END OF QUESTIONS

$$-9 \times 4 = -3 \times (a-3)$$

$$-36 = -3(a-3) \text{ (DIVIDE BY } -3)$$

$$\div (-3) \quad \div (-3)$$

$$12 = a-3$$

$$+3 \quad +3$$

$$15 = a$$

There are no questions printed on this page

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**

Copyright Information

For confidentiality purposes, from the November 2015 examination series, acknowledgements of third party copyright material will be published in a separate booklet rather than including them on the examination paper or support materials. This booklet is published after each examination series and is available for free download from www.aqa.org.uk after the live examination series.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team, AQA, Stag Hill House, Guildford, GU2 7XJ.

Copyright © 2016 AQA and its licensors. All rights reserved.