

Year 11

Name..... SOLUTIONS

Class:

Teacher.....

Higher

Calculator paper

January 2017

Time allowed: 1 Hour

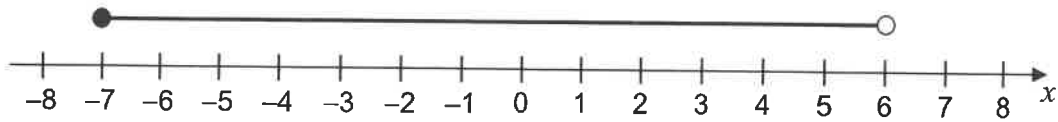
Maximum marks: 52

Answer **all** questions in the spaces provided.

- 1 Circle the inequality shown by the diagram.

SHADED MEANS "OR EQUAL TO"

[1 mark]



$-7 < x < 6$

$-7 \leq x < 6$

$-7 < x \leq 6$

$-7 \leq x \leq 6$

- 2 The probability that a biased coin lands on heads is $\frac{2}{3}$

The coin is spun twice.

Circle the probability of two heads.

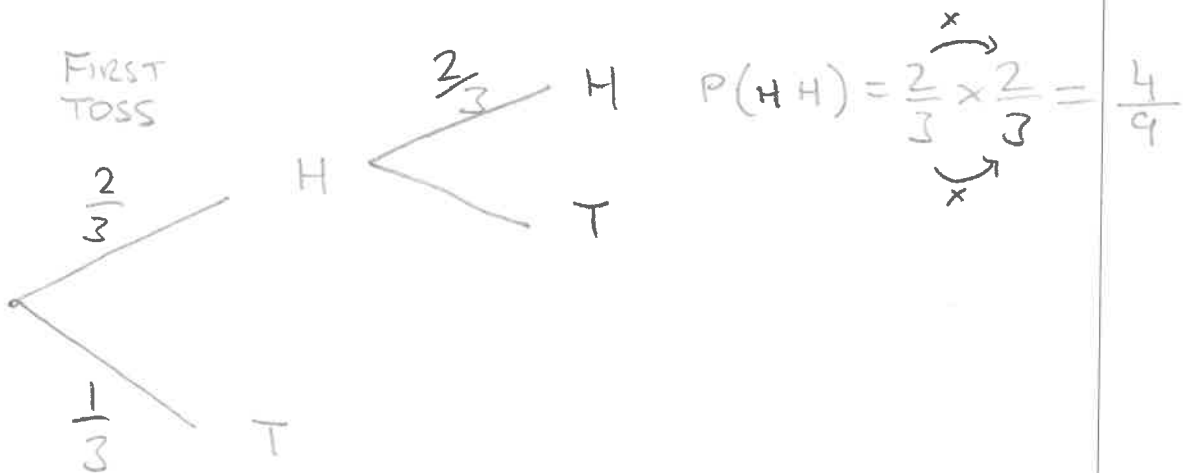
[1 mark]

$\frac{2}{9}$

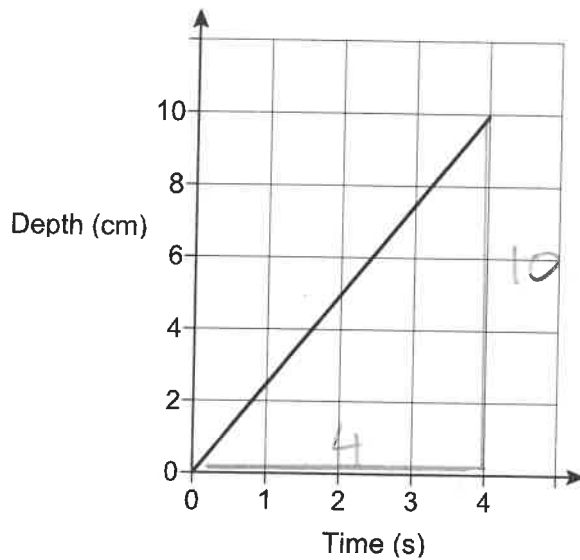
$\frac{4}{6}$

$\frac{4}{9}$

$\frac{4}{3}$



- 3 Water is poured into a glass for 4 seconds.
The graph shows the depth of the water in the glass.



What is the rate of change of the depth of the water?

Circle your answer.

[1 mark]

0.4 cm/s

1.25 cm/s

2.5 cm/s

10 cm/s

$$\text{RATE OF CHANGE} = \text{GRADIENT} = \frac{\text{CHANGE IN } y}{\text{CHANGE IN } x} = \frac{10}{4} = 2.5$$

- 4 The ratio of $x:y$ is $2:3$

2 PARTS : 3 PARTS TOTAL 5 PARTS

Circle the correct statement.

[1 mark]

x is $\frac{2}{3}$ of y

y is $\frac{2}{3}$ of x

x is $\frac{2}{5}$ of y

y is $\frac{3}{5}$ of x

IF y IS 3 PARTS, AND x IS ONLY 2 PARTS

x IS $\frac{2}{3}$ OF y

5 Factorise fully $9a^2 - 6a$

$$= (3) \times 3 \times (a) \times a - (3) \times 2 \times (a)$$

[2 marks]

$$= 3a(3a - 2)$$

COMMON FACTORS
ARE 3 AND a

Answer 3a(3a - 2)

6 Work out the next term of this quadratic sequence.

[2 marks]

	4		12		24		40		<u>60</u>
	⌒		⌒		⌒		⌒		
<u>1st DIFFERENCE</u>	+ 8		+ 12		+ 16		+ 20		
	⌒		⌒		⌒				
<u>2nd DIFFERENCE</u>		+ 4		+ 4		+ 4			

Answer 60

7 Here is an ordinary dice.



7 (a) Ali is going to throw the dice six times.

He says,

"I will get one of each number."

Give a reason why he could be wrong.

[1 mark]

THEORETICAL PROBABILITY SAYS $\frac{1}{6}$ FOR EACH NUMBER,
BUT ACTUAL OR EXPERIMENTAL PROBABILITIES DO
NOT ALWAYS MATCH THEORETICAL PROBABILITIES.

7 (b) Lucy throws the dice 50 times.

Her results are shown.

Number thrown	1	2	3	4	5	6
Frequency	7	4	12	5	9	13

Work out the relative frequency of throwing an odd number.

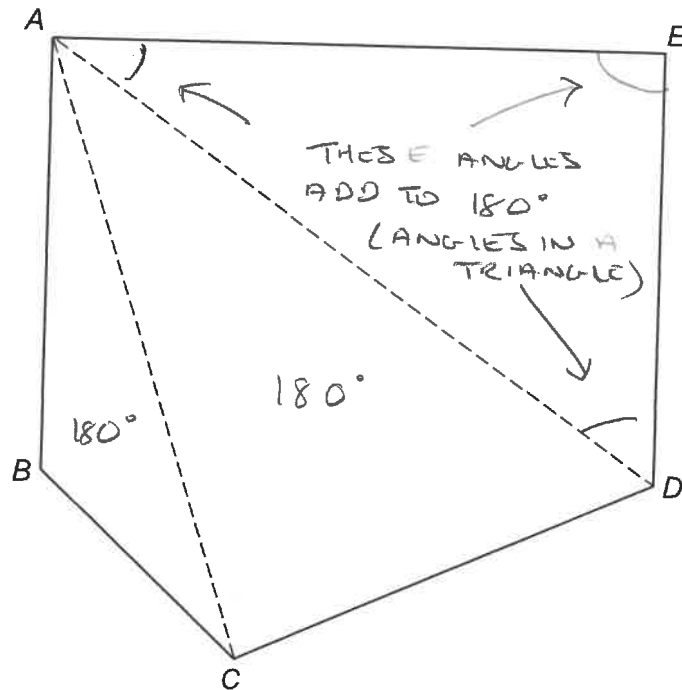
[2 marks]

TOTAL NUMBER OF 'ODD' THROWS = $7 + 12 + 9 = 28$.
SO RELATIVE FREQUENCY IS 28 OUT OF 50 THROWS

$$\text{Answer } \frac{28}{50} = \frac{14}{25} = 0.56$$

8

Polygon $ABCDE$ is divided into triangles as shown.



Not drawn accurately

Use the triangles to work out the sum of the interior angles of polygon $ABCDE$.

You **must** show your working.

[2 marks]

INTERIOR ANGLES ARE THE CORNER ANGLES ON THE INSIDE OF A POLYGON.

THE INTERIOR ANGLES OF EACH OF THE THREE ANGLES ABOVE ARE 180°

SO $180^\circ \times 3$ TRIANGLES = 540°

Answer 540° degrees

9

In a school, 60% of the students are girls. \rightarrow so Boys = 40%.

50% of the girls walk to school. $= \frac{1}{2} = 0.5$

20% of the boys walk to school. $= \frac{1}{5} = 0.2$

What percentage of the students walk to school?

[3 marks]

$$\text{Girls } 60\% \times 0.5 = 30\%$$

$$\text{Boys } 40\% \times 0.2 = 8\%$$

$$\text{Total} = 38\%$$

Answer 38 %

Turn over for the next question

ALTERNATIVE:

ASSUME 100 STUDENTS.

60% GIRLS = 60 GIRLS so 40 BOYS

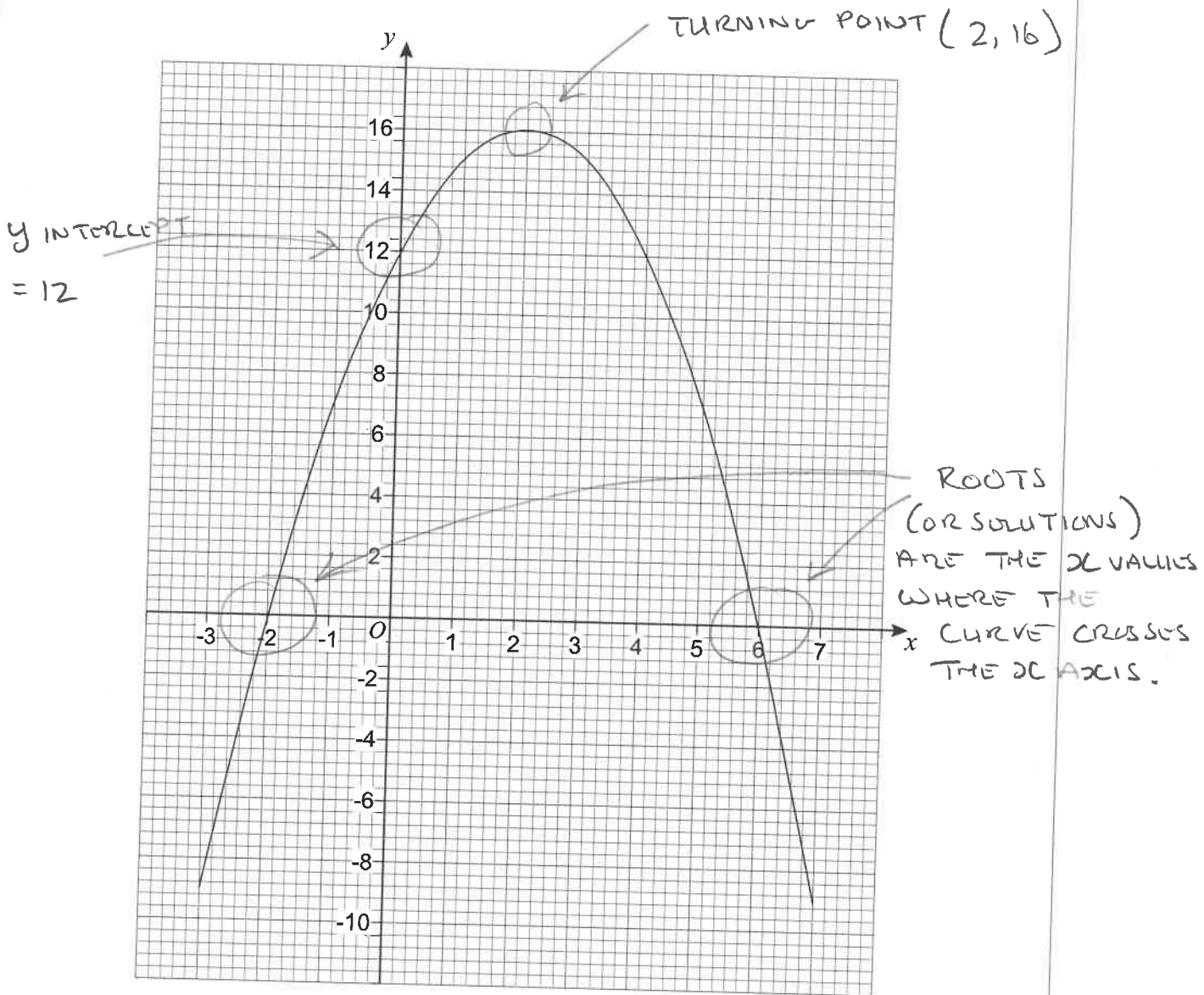
$\frac{1}{2}$ OF 60 GIRLS = 30 GIRLS

$\frac{1}{5}$ OF 40 BOYS = 8 BOYS

$$30 + 8 = 38$$

38 OUT OF 100 STUDENTS = 38%

10 The graph $y = a + bx - x^2$ is shown.



10 (a) Circle the coordinates of the turning point of the curve.

[1 mark]

(-2, 0)

(0, 12)

(2, 16)

(6, 0)

10 (b) Circle the value of a . a IS THE y INTERCEPT IN QUADRATIC GRAPH

[1 mark]

-2

12

16

6

10 (c) Circle the two roots of $a + bx - x^2 = 0$

(WHERE CURVE CROSSED X AXIS)

[1 mark]

-2 and 6

2 and -6

2 and 6

-2 and -6

11 Adam and six other men ran a race.

The times, in seconds, of the six other men are shown.

$$9.75 + 9.79 + 9.80 + 9.88 + 9.94 + 9.98 = 59.14$$

The mean time for **all** seven men was 9.83 seconds.

Did Adam win the race?

You **must** show your working.

[3 marks]

LET THE MISSING TIME BE CALLED x .

THE 'MEAN TIME' = $9.83 = \frac{\text{TOTAL OF SEVEN TIMES}}{7}$

SO TOTAL OF SEVEN TIMES = $7 \times 9.83 = 68.81$

$68.81 = x + 9.75 + 9.79 + 9.80 + 9.88 + 9.94 + 9.98$

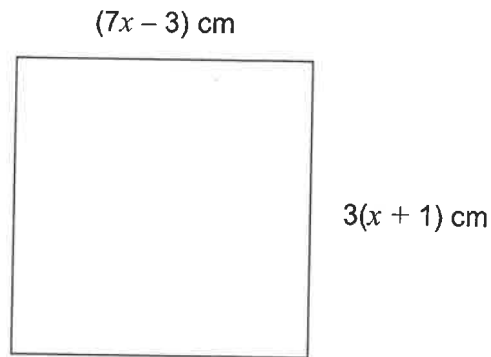
$68.81 = x + 59.14$

$68.81 - 59.14 = x$

$9.67 = x$

So YES, WITH A TIME OF 9.67 SECS.

- 12 The diagram shows a square.



Work out the length of one side of the square.

[4 marks]

AS IT IS A SQUARE, EACH SIDE EQUALS THE OTHER.

So: $7x - 3 = 3(x + 1)$ (EXPAND BRACKETS)

$$\begin{array}{r} 7x - 3 = 3x + 3 \\ -3x \quad -3x \end{array}$$

$$\begin{array}{r} 4x - 3 = 3 \\ +3 \quad +3 \end{array}$$

$$\begin{array}{r} 4x = 6 \\ \div 4 \quad \div 4 \end{array}$$

$$x = \frac{6}{4} = 1.5$$

Answer = 7.5 cm

$$\text{SIDE OF SQUARE} = 3(x + 1) = 3(1.5 + 1) = 3 \times 2.5 = 7.5$$

- 13 A circle has equation $x^2 + y^2 = 4$

Circle the length of its radius.

[1 mark]



4

8

16

IN EQUATION OF CIRCLE WITH CENTRE AT (0,0)

$$x^2 + y^2 = r^2, \text{ so } r^2 = 4 \text{ ABOVE}$$

$$r = \sqrt{4} = 2$$

- 14 a , b and c are different prime numbers less than 20

$$a = \sqrt{4b+c}$$

PRIME NUMBERS LESS THAN 20 ARE:

2, 3, 5, 7, 11, 13, 17, 19

SQUARE THESE: 4, 9, 25, 49, 121,

Work out two possible sets of values of a , b and c .

[3 marks]

USE TRIAL AND ERROR:

(BUT NOTICE $a^2 = 4b+c$)

1/ IF $a = 5$, $5 = \sqrt{4b+c}$ so $4b+c$ MUST EQUAL 25

IF $b = 3$, $4 \times 3 = 12$ AND $12 + 13 = 25$

so $a = 5$, $b = 3$, $c = 13$

2/ IF $a = 7$, $7 = \sqrt{4b+c}$, $4b+c = 49$

IF $b = 11$, $4 \times 11 = 44$, so $44 + 5 = 49$

so $a = 7$, $b = 11$, $c = 5$

Set 1 $a = 5$ $b = 3$ $c = 13$

Set 2 $a = 7$ $b = 11$ $c = 5$

ALSO $a = 5$ $b = 2$ $c = 17$

- 15 Simplify fully $(8x^3y^5)^2 = (8x^3y^5) \times (8x^3y^5)$

[2 marks]

$$= 8 \times 8 \times x^3 \times x^3 \times y^5 \times y^5$$

$$= 64 \times x^{(3+3)} \times y^{(5+5)} = 64x^6y^{10}$$

Answer $64x^6y^{10}$

METHOD 2

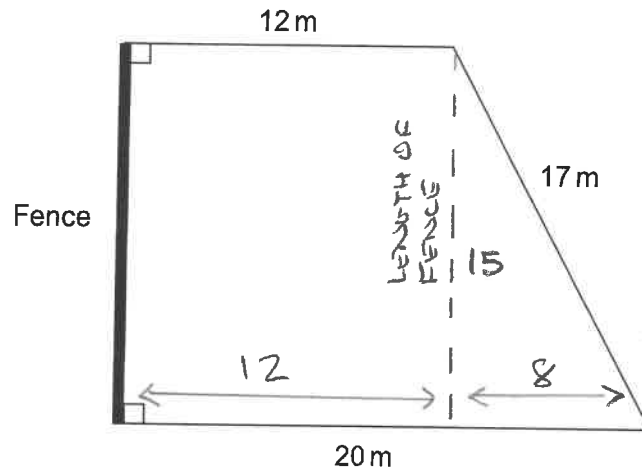
OR $(8x^3y^5)^2 = (8^1x^3y^5)^2$

$$= 8^{1 \times 2} x^{3 \times 2} y^{5 \times 2}$$

$$= 8^2 x^6 y^{10} = 64x^6y^{10}$$

16

The diagram shows a lawn with a fence along one edge.



Not drawn accurately

$$\text{FENCE}^2 = 17^2 - 8^2$$

$$\text{FENCE} = 15$$

One can of weedkiller covers 90 square metres. *

Each can costs £19.25

Work out the total cost of the cans of weedkiller needed to cover the lawn.

[5 marks]

FIND AREA OF SHAPE: NEED LENGTH OF FENCE - USE

PYTHAGORAS' THEOREM

$$\text{FENCE} = \sqrt{17^2 - 8^2} = 15$$

$$\text{AREA OF TRIANGLE} = \frac{1}{2} b \times h = \frac{1}{2} \times 8 \times 15 = 60 \text{ m}^2$$

$$\text{AREA OF RECTANGLE} = b \times h = 12 \times 15 = 180 \text{ m}^2$$

$$\text{TOTAL AREA} = 180 + 60 = 240 \text{ m}^2$$

$$\text{TO FIND HOW MANY CANS NEEDED: } 240 \div 90 = 2.6$$

BUT WE CAN'T BUY 2.6 CANS, MUST BUY 3

$$\text{SO } 3 \times \pounds 19.25 = \pounds 57.75$$

Answer £ 57.75

17 Expand and simplify

$$(2x + 5y)(3x - 8y)$$

$$= 6x^2 - 16xy + 15xy - 40y^2$$

$$= 6x^2 - xy - 40y^2$$

$$2x \times 3x = 6x^2$$

$$2x \times (-8y) = -16xy \text{ [3 marks]}$$

$$5y \times 3x = 15xy$$

$$5y \times (-8y) = -40y^2$$

Answer $6x^2 - xy - 40y^2$

18

The ratio of the number of boys to girls at a party is 3 : 4

Six boys leave the party.

The ratio of the number of boys to girls at the party is now 5 : 8

Work out the number of girls at the party.

[3 marks]

COMPARE MULTIPLES OF THE RATIOS. THE NUMBER OF BOYS MUST BE 6 LESS IN THE SECOND RATIO, BUT NUMBER OF GIRLS MUST BE THE SAME :

B : G	B : G
3 : 4	5 : 8
6 : 8	10 : 16
9 : 12	15 : 24
12 : 16	20 : 32
15 : 20	25 : 40
18 : 24	30 : 48
21 : 28	
24 : 32	
27 : 36	

30 : 40
33 : 44
36 : 48

Answer $30 : 48$, so 48 girls

→ IF 6 BOYS LEAVE, WE ARE LEFT WITH RATIO OF 30 : 48 WHICH MATCHES WITH SECOND RATIO.

19 In the UK in 2000

25% of the population were under 24 years old = LOWER QUANTILE = LQ

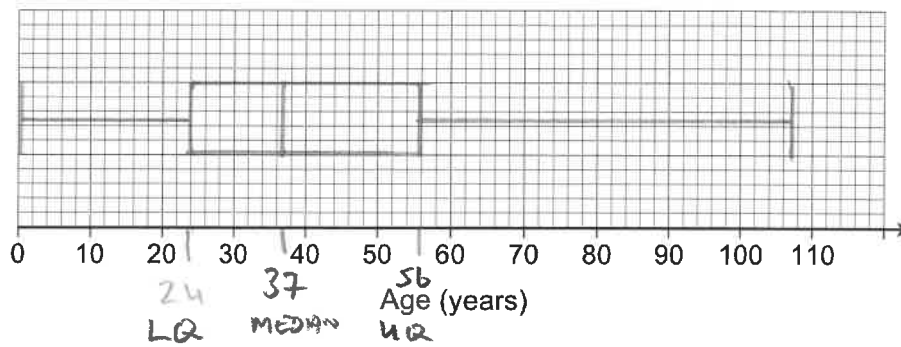
50% of the population were under 37 years old = MEDIAN

the inter-quartile range of the ages was 32 years = IQR

the oldest person was 107 years old.

19 (a) Show the information on a box plot.

[3 marks]



$$\text{INTERQUARTILE RANGE} = \text{UQ} - \text{LQ}$$

$$\text{So: } 32 = \text{UQ} - 24$$

$$32 + 24 = \text{UQ}$$

$$56 \text{ yrs} = \text{UQ}$$

19 (b) It is predicted that in 2050 the age distribution in the UK will have

lower quartile 26 years

median 44 years

upper quartile 66 years

Make **two** comments about the predicted change in the age distribution in the UK from 2000 to 2050

[2 marks]

Comment 1 IF THE MEDIAN GOES UP FROM 37 TO 44
THEN AVERAGE AGE INCREASES ALSO

Comment 2 NEW IQR = $66 - 26 = 40$, SO IQR
RISES BY 8 YEARS, SO VALUES OF AGES ARE
MORE SPREAD OUT

Turn over for the next question

- 20 An amount of money was invested for 8 years.
It earned **compound** interest at 2.5% per year.
After 8 years the total value of the investment was £11 696.67
- 20 (a) Tom is trying to work out the total interest earned.

Tom

$$\text{Interest for 8 years} = £11696.67 \times 0.025 \times 8$$

State what is wrong with Tom's method.

[1 mark]

Tom should "DIVIDE" BY 1.025 TO THE POWER OF 8
SO $11696.67 \div 1.025^8$, TO FIND ORIGINAL AMOUNT.

- 20 (b) Work out the total interest earned.

[3 marks]

$$11696.67 \div 1.025^8 = 9600$$

$$11696.67 - 9600 = £2096.67$$

Answer £ 2096.67

21 Mersenne primes are prime numbers that can be written in the form

$$2^n - 1 \quad \text{where } n \text{ is a whole number.}$$

For example, 3 can be written as $2^2 - 1$

21 (a) Prove that $2^9 - 1$ is **not** a Mersenne prime.

[2 marks]

$2^9 - 1 = 512 - 1 = 511$
 TRY DIVIDING 511 BY DIFFERENT NUMBERS,
 3, 4, 5, 6, 7, 8 etc. ON YOUR CALCULATOR.
 $511 \div 7 = 73$, so 511 is NOT A
 PRIME NUMBER.

21 (b) There are Mersenne primes when $n = 5$ and when $n = 7$

Ama says,

"The ratio of the indices is 5 : 7

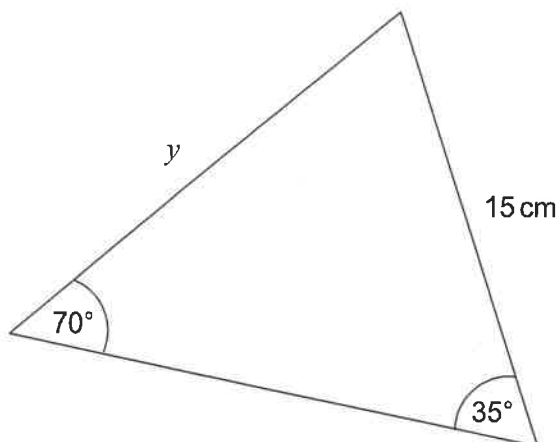
This means the ratio of the Mersenne primes is 5 : 7"

Show that Ama is wrong.

[1 mark]

$2^5 - 1 : 2^7 - 1$
 = $31 : 127$
 THIS WILL NOT SIMPLIFY TO 5:7
 SO AMA IS WRONG.

22

Not drawn
accuratelyWork out the value of y .

[2 marks]

USE SINE RULE:

$$\frac{y}{\sin 35} = \frac{15}{\sin 70} \quad (\text{MULTIPLY BOTH SIDES BY } \sin 35)$$

$$y = \frac{15 \times \sin 35}{\sin 70}$$

$$y = \frac{8.6}{\sin 70}$$

$$y = 9.1558\dots \quad y = 9.16 \text{ (2dp)}$$

Answer 9.16 cm

or 9.2

(or allow 9.15)

23 $2x^2 - 20x + c \equiv a(x - b)^2 + 3b$

Work out the value of c .

$$2x^2 - 20x + c = 2 \left[x^2 - 10x + \frac{c}{2} \right]$$

[3 marks]

COMPLETE THE SQUARE FOR $x^2 - 10x = (x - 5)^2 - (-5)^2$
 $= (x - 5)^2 - 25$

SUBSTITUTE $(x - 5)^2 - 25$ FOR $x^2 - 10x$ ABOVE

$$2 \left[(x - 5)^2 - 25 + \frac{c}{2} \right] \equiv a(x - b)^2 + 3b$$

MULTIPLY OUT

Answer $c = 65$

$$2 \left[(x - 5)^2 - 25 + \frac{c}{2} \right]$$

Turn over for the next question

$$= 2(x - 5)^2 - 50 + c \equiv a(x - b)^2 + 3b$$

So: $a \equiv 2$

$$b \equiv 5$$

$$3b \equiv -50 + c$$

$$3 \times 5 \equiv -50 + c$$

$$15 \equiv -50 + c$$

$$15 + 50 \equiv c$$

$$65 = c$$

2ND METHOD - MULTIPLY OUT

19 AND COMPARE COEFFICIENTS

23

$$2x^2 - 20x + c \equiv a(x-b)^2 + 3b \quad \text{EXPAND } (x-b)^2$$

Work out the value of c .

$$(x-b)(x-b)$$

[3 marks]

$$= x^2 - bx - bx + b^2 = x^2 - 2bx + b^2$$

$$\text{so } a(x-b)^2 = a(x^2 - 2bx + b^2) = ax^2 - 2abx + ab^2$$

$$\text{so } 2x^2 - 20x + c \equiv ax^2 - 2abx + ab^2 + 3b$$

Answer

65

Turn over for the next question

COMPARE COEFFICIENTS ON EACH SIDE

$$2x^2 \equiv ax^2 \quad \text{so } a = 2$$

$$-20x \equiv -2abx \quad \text{AND } a = 2 \text{ so}$$

$$-20x \equiv -2 \times 2 \times b \times x = -4bx$$

$$\text{so } -20 = -4b$$

$$5 = b$$

$$+c \equiv ab^2 + 3b = 2 \times 5^2 + 3 \times 5$$

$$= 50 + 15$$

$$= 65$$

24

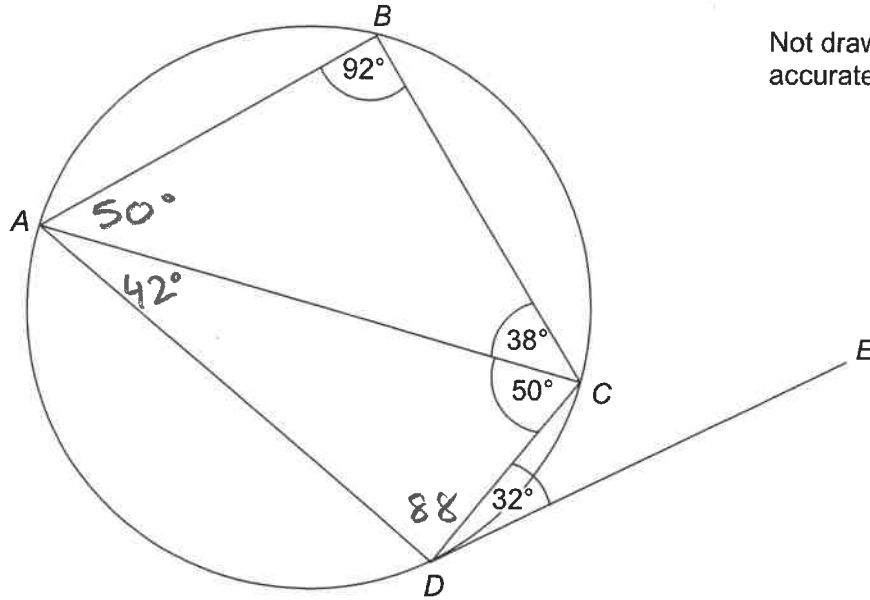
A, B, C and D are points on a circle.

Angle $ABC = 92^\circ$

Angle $ACB = 38^\circ$

Angle $ACD = 50^\circ$

Angle $CDE = 32^\circ$



Not drawn accurately

Tick whether each statement is true or false.

Give a reason for each answer.

[4 marks]

Statement

True

False

AC is a diameter

Reason To be a diameter angle ABC
must equal 90°

Statement

True

False

Angle $ADC = 88^\circ$

Reason $ABCD$ IS A CYCLIC QUADRILATERAL AND OPPOSITE ANGLES IN A CYCLIC QUADRILATERAL ADD TO 180° . $92 + 88 = 180^\circ$

Statement

True

False

 $ABCD$ is a trapezium

Reason $\angle CAB = 180 - 92 - 38 = 50^\circ$
THEREFORE $\angle CAB = \angle ACD = 50^\circ$, SO ALTERNATE ANGLES EQUAL, SO AB IS PARALLEL TO CD

Statement

True

False

 DE is a tangent to the circle

Reason ALTERNATE SEGMENT THEOREM SAYS $\angle CDE = 32^\circ$ MUST EQUAL $\angle CAD = 42^\circ$ WHICH IT DOES NOT.

25

A formula connecting speed (s), distance (d) and time (t) is

$$s = \frac{d}{t}$$

$d = 160$ to 2 significant figures (IN TENS COLUMN)

$t = 7.2$ to 2 significant figures (IN TENTHS COLUMN)

Work out the upper and lower bounds for s .

Give your answers to 3 significant figures.

[4 marks]

$$d = 160 \quad (10 \div 2 = 5) \qquad t = 7.2 \quad (0.1 \div 2 = 0.05)$$

$$\text{MAX } 165 \quad \text{MIN } 155 \qquad \text{MAX } 7.25 \quad \text{MIN } 7.15$$

$$\begin{aligned} \text{LARGEST VALUE OF } S &= \text{BIGGEST } d \div \text{SMALLEST } t \\ &= 165 \div 7.15 = 23.1 \end{aligned}$$

$$\begin{aligned} \text{SMALLEST VALUE OF } S &= \text{SMALLEST } d \div \text{LARGEST } t \\ &= 155 \div 7.25 = 21.4 \end{aligned}$$

Upper bound 23.1 (Allows 23.0...)

Lower bound 21.4 (Allows 21.3...)

26 For all values of x , $f(x) = x^2 + 1$ $g(x) = x - 5$

26 (a) Show that $fg(x) = x^2 - 10x + 26$

[2 marks]

$$\begin{aligned}
 g(x) &= x - 5 & f(x) &= x^2 + 1 & \text{SUBSTITUTE } x-5 \\
 \text{INTO } f(x) &: & f(x-5) &= (x-5)^2 + 1 \\
 & & &= (x-5)(x-5) + 1 \\
 & & &= x^2 - 5x - 5x + 25 + 1 \\
 & & &= x^2 - 10x + 26
 \end{aligned}$$

26 (b) Solve $fg(x) = gf(x)$

[4 marks]

$$\begin{aligned}
 gf(x) &\Rightarrow \text{SUBSTITUTE } x^2+1 \text{ INTO } g(x) = x-5 \\
 g(x^2+1) &= (x^2+1) - 5 = x^2 - 4 \\
 \text{LET } fg(x) &= gf(x)
 \end{aligned}$$

$$\begin{array}{r}
 x^2 - 10x + 26 = x^2 - 4 \\
 -x^2 \qquad \qquad \qquad -x^2
 \end{array}$$

$$\begin{array}{r}
 -10x + 26 = -4 \\
 +4 \qquad \qquad \qquad +4
 \end{array}$$

$$\begin{array}{r}
 -10x + 30 = 0 \\
 +10x \qquad \qquad \qquad +10x
 \end{array}$$

$$x = 3$$

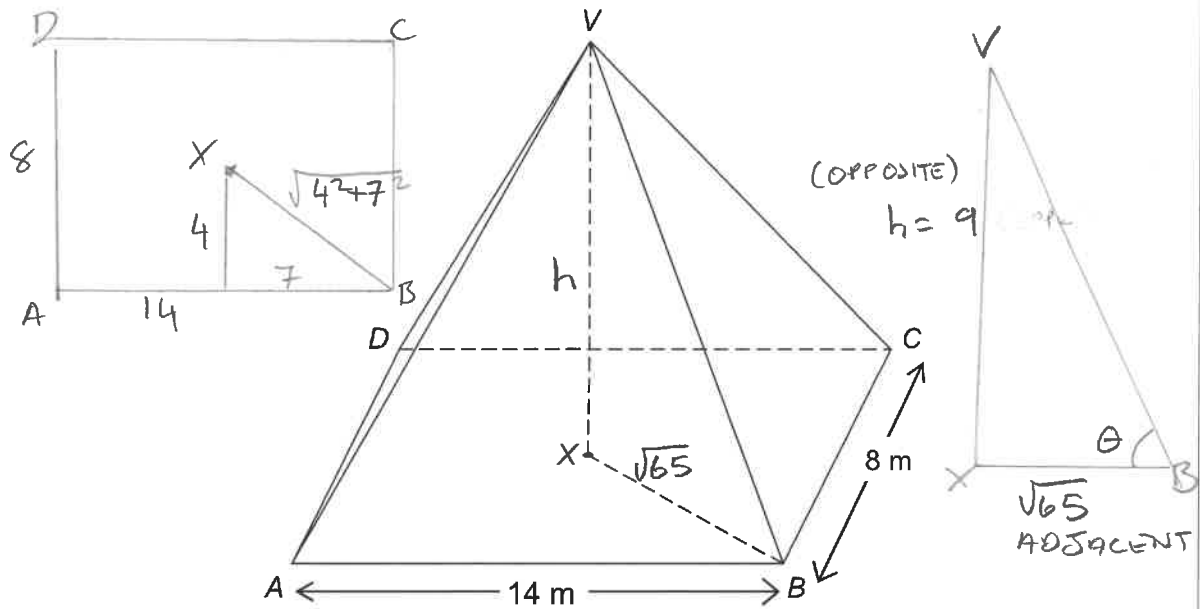
$$30 = 10x$$

$$3 = x$$

27 Volume of a pyramid = $\frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$

VABCD is a rectangular-based pyramid with volume 336 m^3

X is the centre of the horizontal base, directly below V.



Work out the angle between VB and the base.

USING PYTHAGORAS

[6 marks]

$$XB = \sqrt{4^2 + 7^2} = \sqrt{16 + 49} = \sqrt{65}$$

$$\text{VOLUME} = \frac{1}{3} \times \text{AREA OF BASE} \times h = 336 \quad \text{AREA OF BASE} = 14 \times 8 = 112$$

$$= \frac{1}{3} \times 112 \times h = 336$$

$$37.3h = 336$$

$$h = 9$$

FOR ANGLE θ : $\tan \theta = \frac{\text{OPPOSITE}}{\text{ADJACENT}} = \frac{9}{\sqrt{65}}$

$$\theta = \tan^{-1} \left(\frac{9}{\sqrt{65}} \right)$$

$$\theta = 48^\circ \text{ (or } 48.1\dots^\circ)$$

Answer 48° (or $48.1\dots$) degrees

END OF QUESTIONS