

Please write clearly in block capitals. *WRITTEN SOLUTIONS*

Centre number

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 Candidate number

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Surname _____

Forename(s) _____

Candidate signature _____

GCSE MATHEMATICS

H

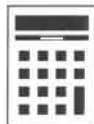
Higher Tier Paper 2 Calculator

Date of Exam Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- a calculator
- mathematical instruments.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

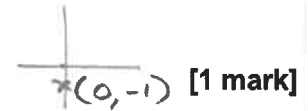
Answer **all** questions in the spaces provided.

1

The equation of a line is

$y = 2x - 1$ → y-INTERCEPT
2 IS GRADIENT

Circle the point that is the intercept with the y-axis.



(0, -1)

(-1, 0)

 $(0, \frac{1}{2})$ $(\frac{1}{2}, 0)$

[1 mark]

2

Simplify

$$\frac{a^4 \times a^6}{a^3} = \frac{a^{4+6}}{a^3} = \frac{a^{10}}{a^3} = a^{10-3} = a^7$$

Circle your answer.

 a^7 a^8 a^{21} a^6

[1 mark]

3

A coin is thrown 50 times.

The coin lands on heads 30 times. =

Circle the relative frequency of landing on heads. = 30 OUT OF TOTAL TIMES

[1 mark]

30

3 : 5

30%

 $\frac{3}{5}$

$$\frac{30}{50} = \frac{3}{5}$$

- 4 A number, x , is 15.8 when rounded to 3 significant figures.

Circle the error interval.

$$\begin{array}{r} 0.1 \\ \frac{1}{10} \\ \text{---} \\ 15.8 \\ \text{---} \\ \uparrow \\ \text{3rd SIGNIFICANT FIGURE} \end{array}$$

$$15.75 < x < 15.85$$

$$15.75 \leq x < 15.85$$

$$15.75 < x \leq 15.85$$

$$15.75 \leq x \leq 15.85$$

[1 mark]

x CAN BE 15.75
BUT CANNOT BE
15.85

$$\begin{array}{l} 15.8 + 0.05 = 15.85 \\ 15.8 - 0.05 = 15.75 \end{array}$$

3rd SIGNIFICANT FIGURE

IS IN $\frac{1}{10}$ COLUMN SO $0.1 \div 2 = 0.05$

BUT x CANNOT
BE 15.85 SO
 $x < 15.85$

- 5 (a) Expand and simplify

$$(x+5)(x-4)$$

$$= x^2 - 4x + 5x - 20$$

$$= x^2 + x - 20$$

[2 marks]

Answer

$$x^2 + x - 20$$

- 5 (b) Solve $(x-8)(x+7)=0$ WHAT VALUES OF x MAKE

THE EQN. ZERO

[1 mark]

$$\text{Let } x = 8$$

$$\text{Let } x = -7$$

$$(8-8)(8+7)$$

$$(-7-8)(-7+7)$$

$$= 0 \times 15 = 0$$

$$= -15 \times 0 = 0$$

Answer

$$x = 8$$

$$x = -7$$

6 Dev invests £1500 for 2 years.

The compound interest rate is 1.6% per year. $MULTIPLIER = \frac{100 + 1.6}{100} = 1.016$

6 (a) Which calculation works out the total value after 2 years?

Circle your answer.

$$1500 \times 1.016 \times 1.016 = 1500 \times 1.016^2$$

[1 mark]

$$£1500 \times 1.6 \times 2$$

$$£1500 \times 1.6^2$$

$$£1500 \times 1.016 \times 2$$

$$£1500 \times 1.016^2$$

6 (b) Emma invests £1500 for 2 years.

The interest rate is

1.8% for the first year

1.3% for the second year.

$$MULTIPLIER = \frac{100 + 1.8}{100} = 1.018$$

$$MULTIPLIER = \frac{100 + 1.3}{100} = 1.013$$

Whose investment is worth more after 2 years?

You must show your working.

[4 marks]

$$EMMA: 1500 \times 1.018 = 1527$$

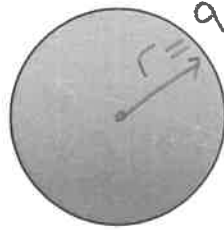
$$1527 \times 1.013 = 1546.85 \text{ (2 d.p.)}$$

$$DEV: 1500 \times 1.016^2 = 1548.38 \text{ (2 d.p.)}$$

Answer DEV

7 Volume of a sphere = $\frac{4}{3}\pi r^3$

A steel sphere, radius 9 cm, is shown.



7 (a) Work out the volume of the sphere.

[2 marks]

$$V = \frac{4}{3} \times \pi \times 9^3 = 972\pi = 3054.1$$

$$= 3050 \text{ (NEAREST TEN)}$$

Answer 972π or 3050 (NEAREST TEN) cm^3
or 3054

7 (b) The density of the steel is 7.8 grams/cm^3

Work out the mass of the sphere.

or $D = \frac{m}{v}$

[2 marks]

$$7.8 = \frac{m}{3054}$$

$$m = 3054 \times 7.8$$

$$= 23821.2$$

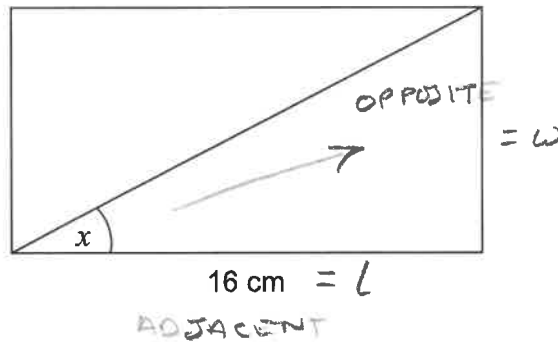
Answer 23821.2 grams

Turn over for the next question

Turn over ►

8 The area of the rectangle is 68 cm^2

Not drawn
accurately



FIND MISSING
SIDE USING
AREA = $L \times W$
 $68 = 16 \times W$

2 USE
SOH
CAH
TOA
 $\text{TAN} = \frac{\text{OPPOSITE}}{\text{ADJACENT}}$

Work out the size of angle x .

[3 marks]

$$\text{AREA} = L \times W = 16 \times W = 68$$

$$W = 68 \div 16 = 4.25$$

$$\text{TAN } x = \frac{4.25}{16}$$

$$x = \text{Tan}^{-1}\left(\frac{4.25}{16}\right) = 14.87$$

Answer 14.87 degrees
or 14.9 etc.

9 Which number is **not** in standard form?

Circle your answer.

✓
 1.01×10^9

0.99×10^{-2}

✓
 9.8×10^6

✓
 4.632×10^{-5}

[1 mark]

STANDARD FORM MUST HAVE A NUMBER
BETWEEN 1 AND 9 INCLUSIVE IN FRONT
OF DECIMAL POINT

10

A charity collection was made.

Information about the amounts given by men is shown in the table.

Amount, x (£)	Midpoint	Number of men	
$0 \leq x < 5$	$(5-0) \div 2 = 2.5$ x	11	27.5
$5 \leq x < 10$	7.5 x	7	52.5
$10 \leq x < 15$	12.5 x	2	25
		Total = 20	105

↑
ADD
↓

The mean amount given by **women** was £6.30 per person.

Compare the mean amounts given by men and women.

[4 marks]

$$\underline{\text{MEAN FOR MEN}} = 105 \div 20 = 5.25$$

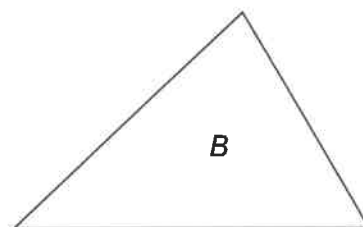
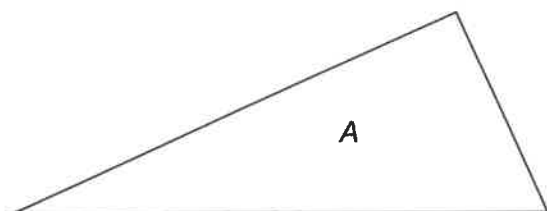
WOMEN GAVE MORE

Turn over for the next question**Turn over ►**

11 The angles in triangle A are in the ratio 1 : 2 : 3

The angles in triangle B are in the ratio 4 : 5 : 6

Not drawn
accurately



Jack says,

"The middle number in each ratio is one third of the total,
so one of the angles in each triangle is 60 degrees"

Is he correct?

Show working to support your answer.

[2 marks]

$$\text{For A: } 1:2:3 \text{ add} = 6 \quad 180^\circ \div 6 = 30^\circ$$

$$\text{So angles for A must be } 1 \times 30 : 2 \times 30 : 3 \times 30 \\ = 30 : 60 : 90$$

$$\text{For B: } 4:5:6 \text{ add} = 15 \quad 180^\circ \div 15 = 12^\circ$$

$$4 \times 12 : 5 \times 12 : 6 \times 12$$

$$48^\circ : 60^\circ : 72^\circ$$

MIDDLE NUMBER IS 60° SO YES

- 12 In a class, the ratio boys : girls is $x : y$ GIRLS = y

Circle the fraction of the class that are girls.

[1 mark]

$$\frac{x}{y}$$

$$\frac{y}{x}$$

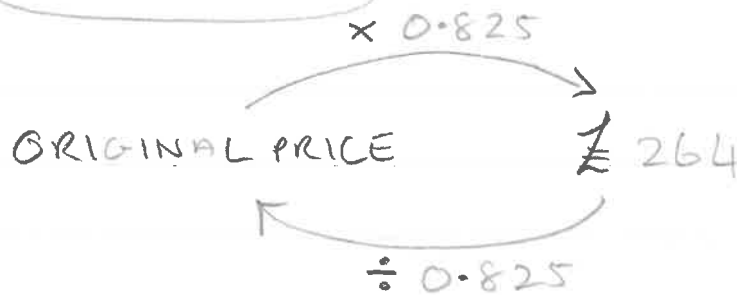
$$\frac{x}{x+y}$$

$$\frac{y}{x+y}$$

TOTAL IN CLASS = $x + y$
FRACTION OF GIRLS MUST BE $\frac{y}{x+y}$

- 13 The price of a computer is reduced by 17.5% MULTIPLIER = $\frac{100 - 17.5}{100}$
The reduced price is £264
By how much is the price reduced? = 0.825

[4 marks]



TO FIND ORIGINAL AMOUNT : $264 \div 0.825$
= 320

REDUCTION = $320 - 264 = 56$

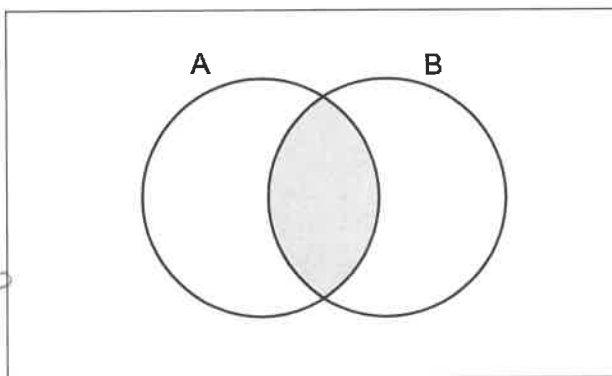
Answer £

56

- 14 ξ = numbers between 0 and 1 with 3 decimal places
 A = numbers that round to 0.7 to 1 decimal place
 B = numbers that round to 0.75 to 2 decimal places

14 (a) Work out a possible number, with 3 decimal places, that is in the shaded area.

A
 NUMBERS THAT ROUND TO 0.7 TO 1dp
 0.65, 0.66, 0.67
 0.68, 0.69, 0.70
 0.71, 0.72, 0.73
 0.74



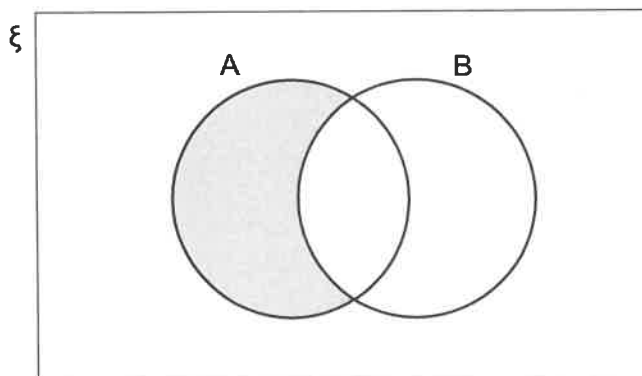
B
 NUMBERS THAT ROUND TO 0.75 TO 2dp
 0.750, 0.751, 0.752
 0.753, 0.754,
 0.745, 0.746
 0.747, 0.748
 0.749

[1 mark]

0.745, 0.746, 0.747, 0.748, 0.749
 ARE 3dp NUMBERS THAT ROUND TO 0.7 TO 1dp
 AND 0.75 TO 2dp

Answer AS ABOVE

14 (b) Work out a possible number, with 3 decimal places, that is in the shaded area.

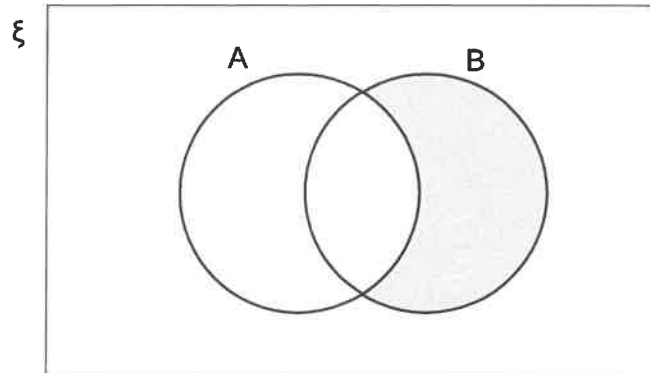


[1 mark]

Any 3dp NUMBER FROM 0.650 TO 0.744
 WILL ONLY ROUND TO 0.7 TO 1dp BUT NOT
 0.75 TO 2dp.

Answer eg 0.651

- 14 (c) Work out a possible number, with 3 decimal places, that is in the shaded area.

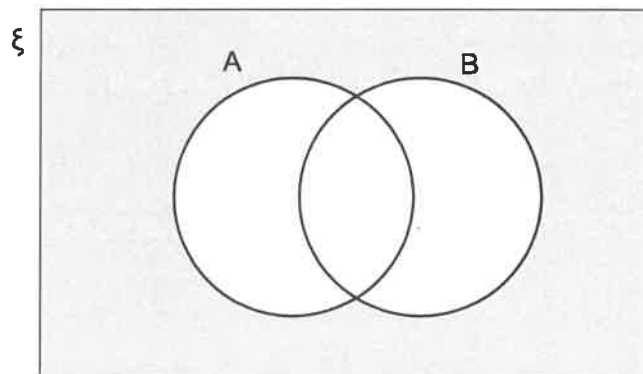


[1 mark]

B: Numbers from 0.750 to 0.754

Answer eg 0.751

- 14 (d) Work out a possible number, with 3 decimal places, that is in the shaded area.



[1 mark]

Between 0 and 1

eg. 0.001 WILL NOT BE IN A OR B

Answer 0.001

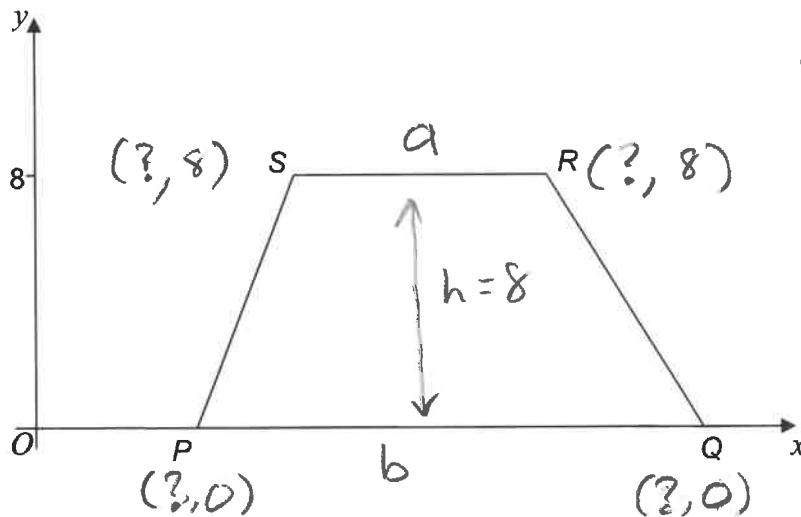
Turn over ►

15 PQRS is a trapezium with PQ parallel to SR.

P and Q are on the x-axis.

The y-coordinate of S is 8

$PQ > SR$



Not drawn
accurately

The area of PQRS is 48 square units.

Work out one possible set of points for P, Q, R and S.

[4 marks]

$$\text{Area of trapezium} = \frac{1}{2} (a+b) \times h$$

$$\text{so } 48 = \frac{1}{2} \times (a+b) \times 8 = 4(a+b)$$

$$48 = 4(a+b) \quad (\text{Divide both sides by } 4)$$

$$12 = a+b$$

b is bigger than a so a could be 4

and b could be 8 (or a = 5, b = 7 etc)

So difference eg P (2 , 0) (2 + 8 = 10)

between P + Q Q (10 , 0)

∴ coordinates R (5 , 8) MAKE R SAY S = x

could be 8,

$$5 + 4 = 9$$

and S and R

S (9 , 8)

could be 4

- 16 (a)** I am thinking of two different numbers.
They are both greater than 10
Their highest common factor (HCF) is half the smaller number.
Work out one possible pair of numbers.

[1 mark]

TRY : 16 24
FACTORS: 1, 2, 4, (8), 16 1, 2, 3, 4, (8), 12, 24
HCF = 8

Answer 16 and 24
or: 12 18
or 100 250 etc

- 16 (b)** a , b and c are prime numbers.

$$N = a^3 \times b^2 \times c$$

a could be 2, or b could be 2
or c could be 2 : see below

Is N always odd?

Tick a box.

Yes

No

Give reasons for your answer.

[2 marks]

PRIME CAN BE ODD OR EVEN (2)
ODD \times ODD = ODD EVEN \times EVEN = EVEN
EVEN \times ODD = EVEN LET $C=2$ (EVEN)
THEREFORE a^3 CAN BE ODD IF a IS ODD
 $b^2 = \text{ODD} \times \text{ODD} = \text{ODD}$
 $a^3 \times b^2 = \text{ODD} \times \text{ODD} = \text{ODD}$
 $a^3 \times b^2 \times c = \text{ODD} \times \text{ODD} \times \text{EVEN} = \text{ODD} \times \text{EVEN}$
= EVEN

SO NOT ALWAYS ODD

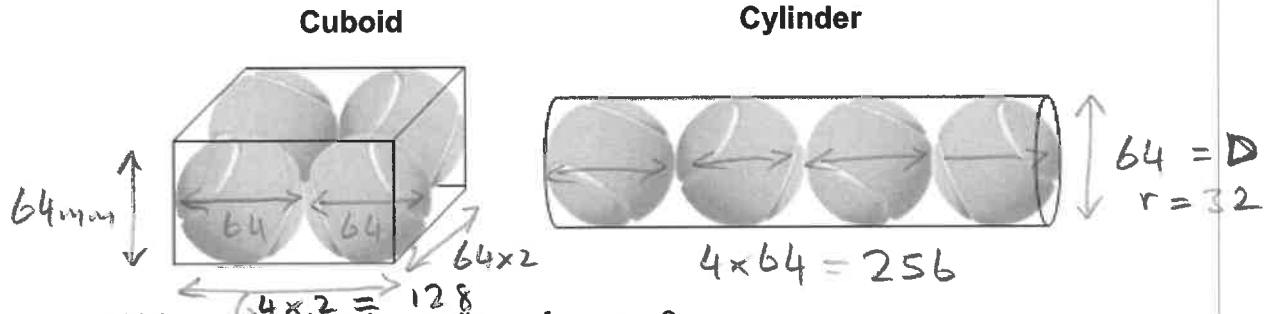
Turn over ►

17

Here are two closed containers.

Four tennis balls just fit in each container.

Each tennis ball has diameter 64 mm



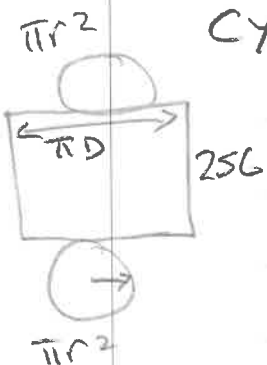
Which container has the smaller surface area?

You **must** show your working.**[5 marks]**CUBOID MUST HAVE DIMENSIONS $h = 64$, $w = 64 \times 2 = 128$ AND DEPTH = $64 \times 2 = 128$ SO SURFACE AREA: FRONT AND BACK : $64 \times 128 + 64 \times 128$
 $= 16384$ TWO SIDES = $128 \times 64 + 128 \times 64 = 16384$ TOP + BOTTOM = $128 \times 128 + 128 \times 128 = 32768$ TOTAL = $32768 + 16384 + 16384 = 65536$ CYLINDER : 2 CIRCULAR ENDS : AREA = $\pi r^2 \times 2$
 $= \pi \times 32^2 \times 2 = 2048\pi$ AREA OF BODY = CIRCUMFERENCE \times 256 $= \pi \times 64 \times 256 = 16384\pi$ TOTAL = $2048\pi + 16384\pi = 18432\pi$ $= 57929.142$

CYLINDER HAS SMALLER SURFACE AREA

Answer

CYLINDER



18 Here is some information about the times, in minutes, 80 teachers took to get to work.

Time t (minutes)	Frequency	CUMULATIVE FREQUENCY
$0 < t \leq 20$	12	12
$20 < t \leq 40$	32	$12 + 32 = 44$
$40 < t \leq 60$	25	$44 + 25 = 69$
$60 < t \leq 90$	11	$69 + 11 = 80$

18 (a) On the grid, draw a cumulative frequency graph.

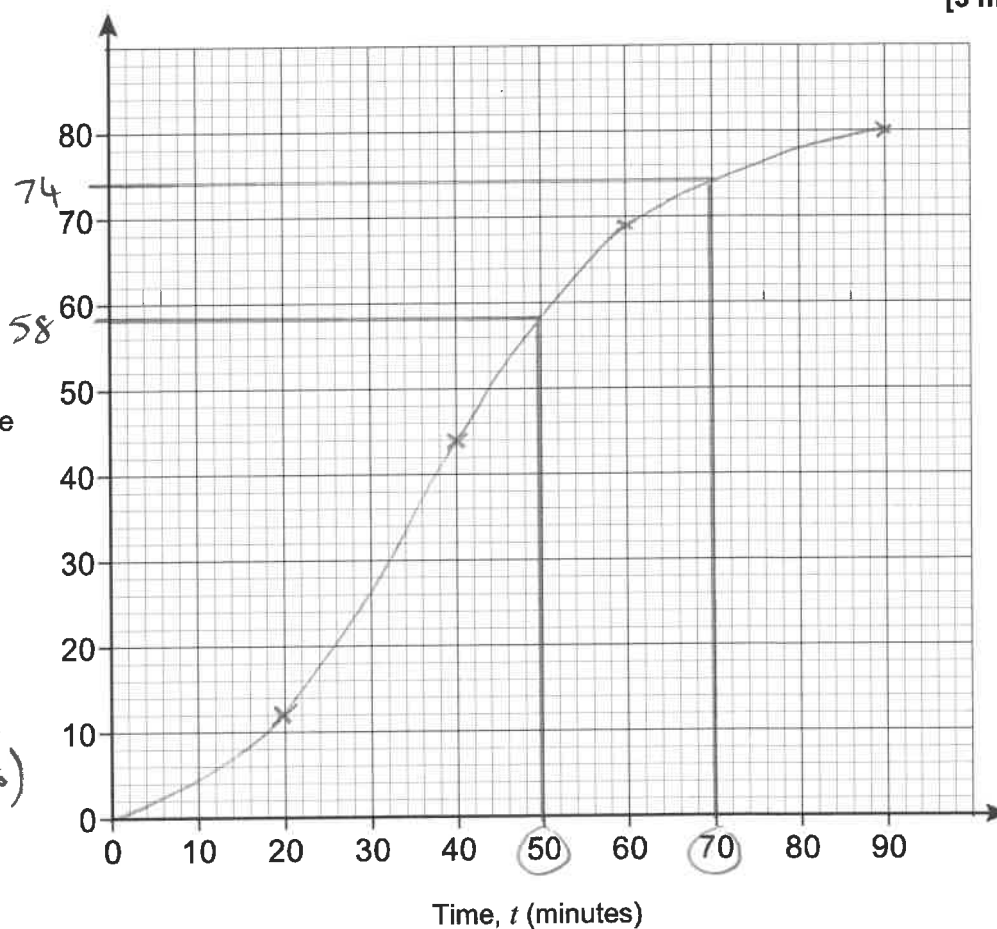
[3 marks]

PLOT POINTS AT UPPER END OF CLASS INTERVALS:

- (20, 12)
- (40, 44)
- (60, 69)
- (90, 80)

Cumulative frequency

(SMALL SQUARES GOING UP IN TWO'S)



DRAW CURVED OR STRAIGHT LINES TO JOIN POINTS

18 (b) Estimate the number of teachers who took between 50 minutes and 70 minutes to travel to work.

[2 marks]

AT 50 MINS : APPROXIMATELY 58

AT 70 MINS : APPROX : 74

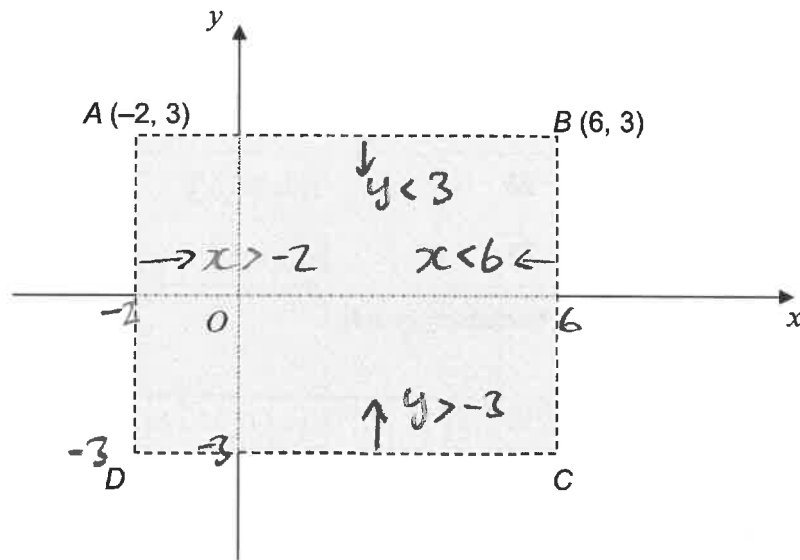
$$74 - 58 = 16$$

Answer 16

ANSWERS AROUND 15, 16, 17

Turn over ►

- 19 (a) $ABCD$ is a rectangle.
The x -axis is a line of symmetry.



These inequalities describe the shaded region.

$$p < x < q \text{ and } r < y < s$$

Write down the values of p , q , r and s .

[2 marks]

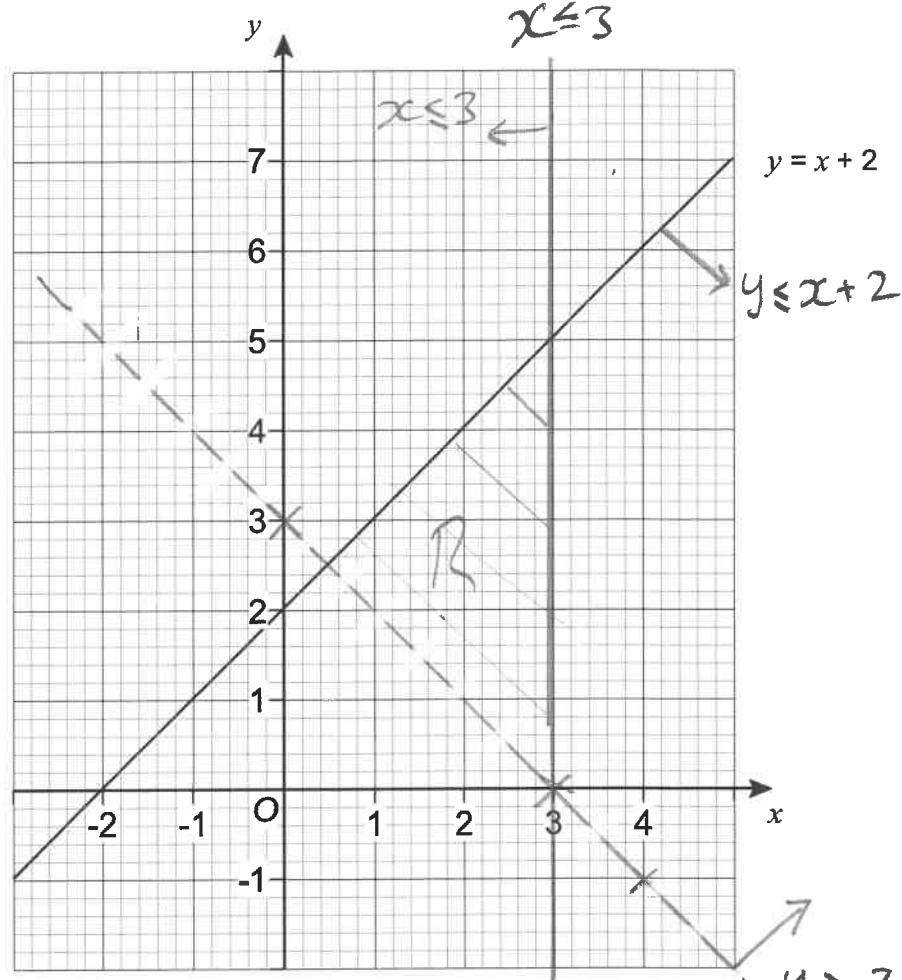
$$p = -2$$

$$q = 6$$

$$r = -3$$

$$s = 3$$

19 (b) The grid shows the graph of $y = x + 2$



FOR $y > 3 - x$,
SELECT VALUES OF x
AND SUBSTITUTE
INTO $y = 3 - x$

x	0	3	
y	3	0	

$x = 0$
 $y = 3 - 0 = 3$
 $(0, 3)$
 $x = 3$
 $y = 3 - 3 = 0$
 $(3, 0)$
 $x = 4$
 $y = 3 - 4 = -1$
 $(4, -1)$

PLT POINTS
BUT $y > 3 - x$
SO AREA
ABOVE THE
LINE

(DOTTED LINE
AS $y >$ NOT $y \geq$)

On the grid, identify the region represented by

$y \leq x + 2$ and $y > 3 - x$ and $x \leq 3$
Label the region R.
LESS THAN MEANS 'UNDER' THE LINE
(DRAW LINE $x = 3$)
[3 marks]

AS $x \leq 3$ IT MUST BE
A SOLID LINE.

Turn over for the next question

Turn over ►

20 $f(x) = 3^{2x}$ and $g(x) = x^3$ for all values of x .

20 (a) Work out the value of $f(1) + g(4)$

[2 marks]

$$f(1) = 3^{2 \times 1} = 3^2 = 9 \quad g(4) = 4^3 = 64$$

$$9 + 64 = 73$$

Answer 73

20 (b) Work out the value of $g^{-1}(-27)$

[2 marks]

$$g^{-1}(x) \text{ EQUALS INVERSE OF } x^3 = \sqrt[3]{x}$$

$$g^{-1}(-27) = \sqrt[3]{-27} = -3$$

(BECAUSE $-3 \times -3 \times -3 = (-3)^3 = -27$)

Answer -3

20 (c) Work out an expression for $gf(x)$

Give your answer as a power of 3 in its simplest form.

[2 marks]

$$f(x) = 3^{2x} \quad g(x) = x^3$$

$$\text{so } gf(x) = g(3^{2x}) = (3^{2x})^3 = 3^{2x \times 3}$$

$$= 3^{6x}$$

Answer 3^{6x}

20 (d) One of these graphs is a sketch of $y = 3^{2x}$ EXPONENTIAL GRAPH
Which one?

Circle the correct letter.

TEST SOME VALUES

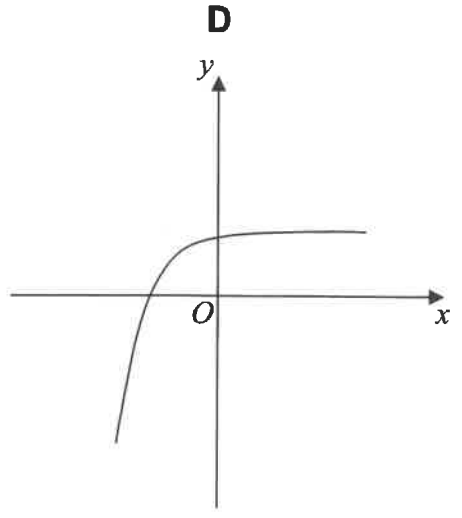
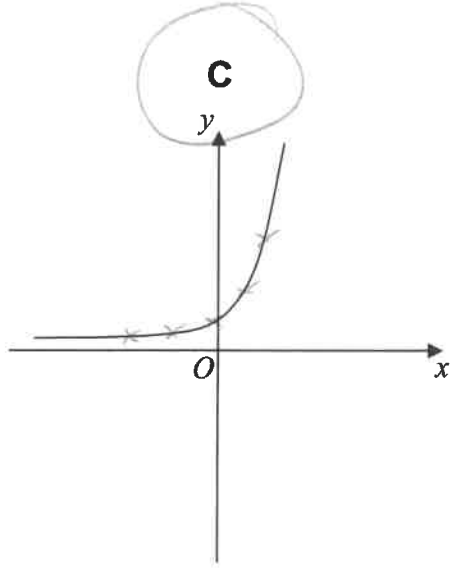
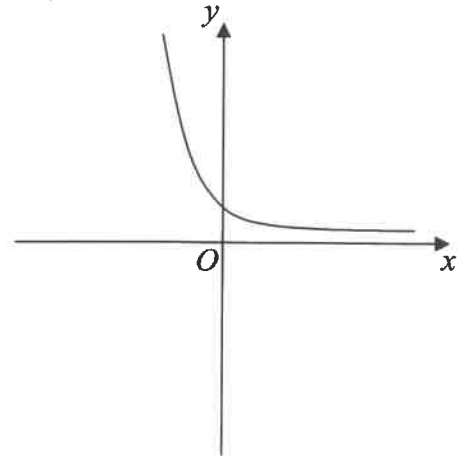
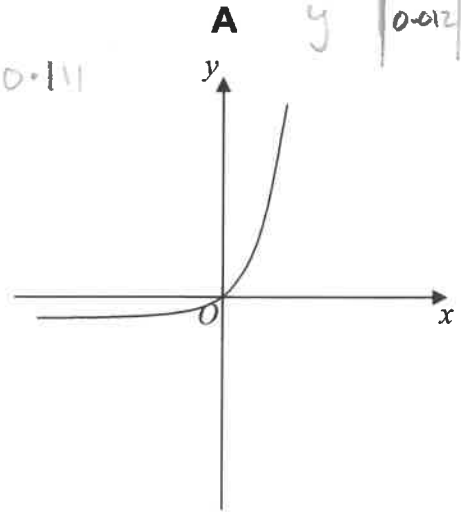
x	-2	-1	0	1	2	3	4
3^{2x}	3^{-4}	3^{-2}	$3^0 = 1$	3^2	3^4		
y	0.012	0.111	1	9	81		

[1 mark]

SO GRAPH C

$$3^{-4} = \frac{1}{3^4} = 0.012$$

$$3^{-2} = \frac{1}{9} = 0.111$$



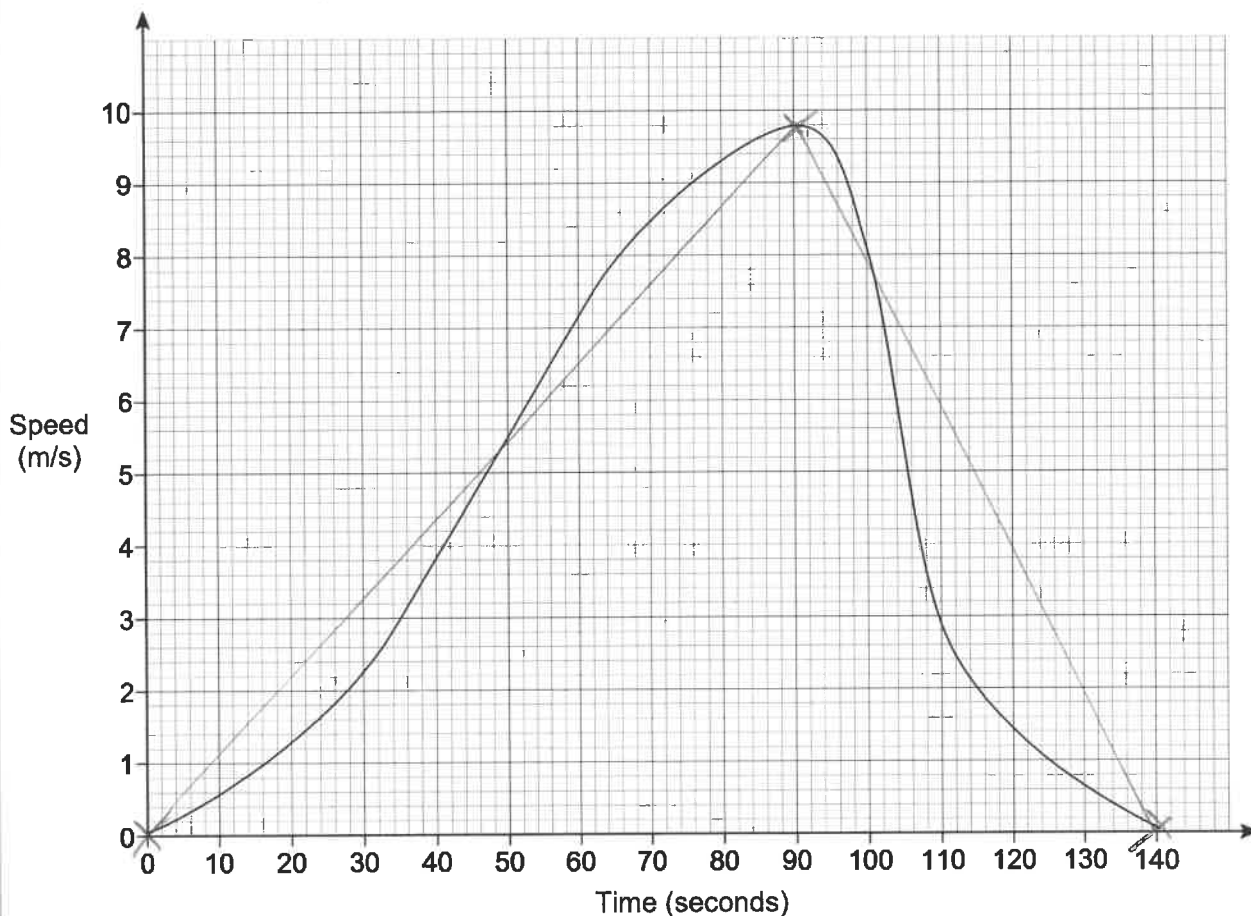
Turn over for the next question

Turn over ►

21 The graph shows the speed of a skier.

Nick wants to estimate the distance travelled by the skier in 140 seconds.

AREA UNDER SPEED-TIME GRAPH GIVES DISTANCE TRAVELLED.



21 (a) He works out the area of the triangle with vertices $(0, 0)$, $(140, 0)$ and $(90, 9.8)$

Does Nick's method give a good estimate?

Tick a box.

Yes



No



Give a reason for your answer.

[2 marks]

AN ESTIMATE OF THE AREA UNDER THE GRAPH GIVES THE DISTANCE TRAVELLED, AND THIS CAN BE FOUND BY FINDING THE AREA UNDER THE TRIANGLE. THE EXTRA AREAS ABOVE AND BELOW THE SIDES OF THE TRIANGLE 'APPROXIMATELY' CANCEL EACH OTHER OUT.

21 (b) Use Nick's method to work out an estimate of the distance.

[1 mark]

$$\begin{aligned} \text{AREA OF TRIANGLE} &= \frac{1}{2} \times \text{BASE} \times \text{HEIGHT} \\ &= \frac{1}{2} \times 140 \times 9.8 = 686 \end{aligned}$$

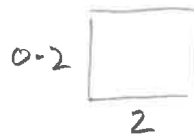
Answer 686 metres

21 (c) Helen uses a different method.

She starts by estimating how many of the smallest squares are in the region between the graph and the horizontal axis.

Her estimate is 1550 squares.

Complete Helen's method to estimate the distance.



[2 marks]

$$\begin{aligned} \text{AREA OF EACH SMALL SQUARE IS } & 0.2 \times 2 = 0.4 \\ \text{SO TOTAL} &= 1550 \times 0.4 = 620 \end{aligned}$$

Answer 620 metres

Turn over for the next question

Turn over ►

22

The value of a car, £ V , after t years, is modelled by the equation

$$V = A \times k^{-t} \quad \text{where } A \text{ and } k \text{ are constants.}$$

WHEN NEW TIME
EQUALS ZERO

The value of the car when new was £22 000 $V = 22000$ when $t = 0$

The value of the car after 2 years is £14 080 $V = 14080$ when $t = 2$

Work out the values of A and k .

[4 marks]

$$V = A \times k^{-t}$$

TO FIND A:

SUBSTITUTE IN $V = 22000$, $t = 0$

$$22000 = A \times k^0 = A$$

$$\text{so } V = 22000 k^{-t} \quad \left(\text{SUBSTITUTE IN } V = 14080 \right.$$

$$14080 = 22000 \times k^{-2} \quad \left. \text{AND } t = 2 \text{ TO FIND } k \right)$$

$$14080 = \frac{22000}{k}$$

$$k = \frac{22000}{14080} = 1.25$$

$$A = 22000$$

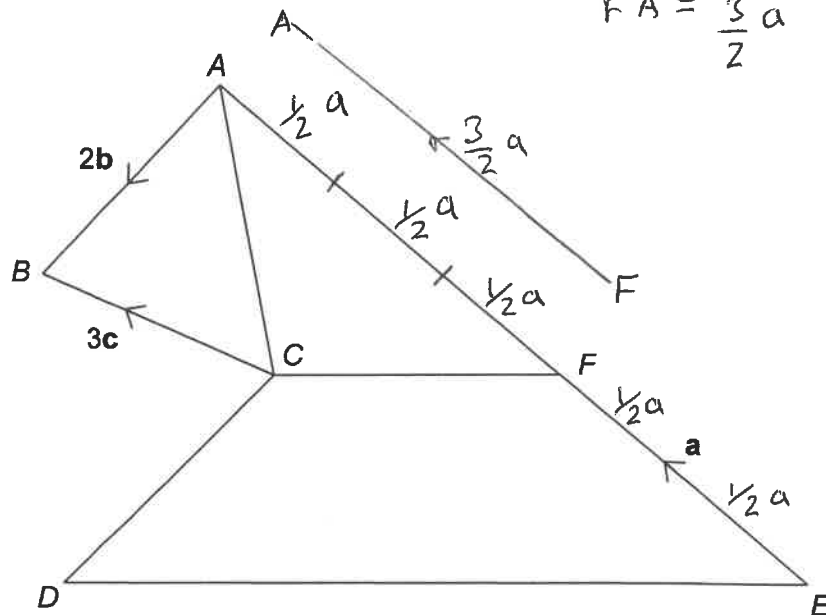
$$k = 1.25$$

23

 AFE is a straight line. $AF : FE = 3 : 2$ so $\frac{3}{5}$ AND $\frac{2}{5}$, so $a = \frac{2}{5}$ DE is parallel to CF . $DE = 2CF$ $\vec{EF} = a$ $\vec{AB} = 2b$ $\vec{CB} = 3c$

$$\frac{1}{2}a = \frac{1}{5}$$

$$FA = \frac{3}{2}a$$

Work out \vec{DE} in terms of a , b and c .**[4 marks]**

$$DE = 2CF$$

$$CF = \vec{CB} + \vec{BA} + \vec{AF}$$

$$= 3c - 2b - \frac{3}{2}a$$

$$2CF = 2(3c - 2b - \frac{3}{2}a)$$

$$= 6c - 4b - 3a$$

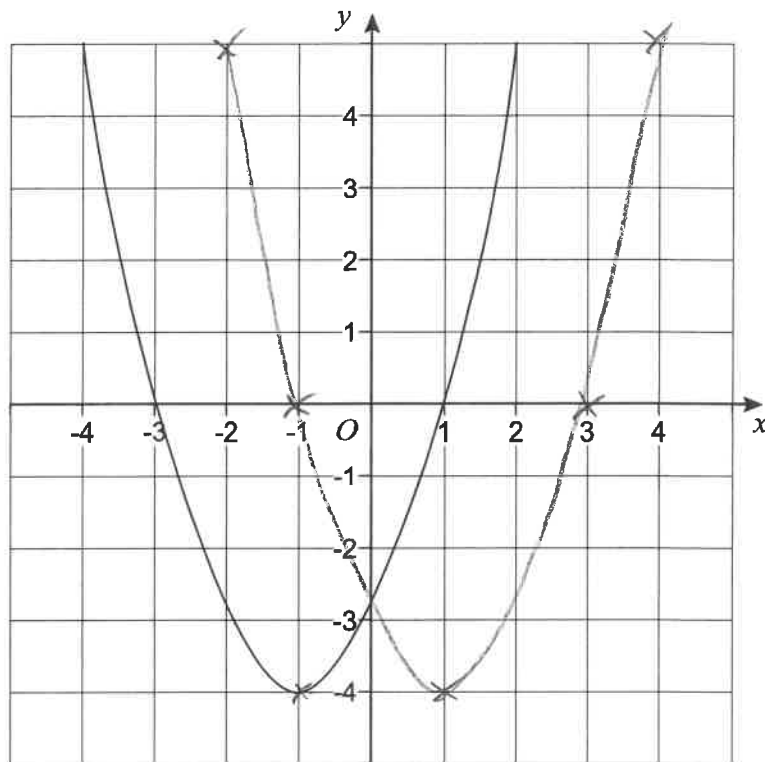
So

$$DE = 2CF = 6c - 4b - 3a$$

Answer

Turn over ►

- 24 (a) Here is the graph of $y = f(x)$
The graph has a turning point at $(-1, -4)$



On the grid, draw the graph of $y = f(x - 2)$

[1 mark]

THIS TRANSLATES THE GRAPH,
TWO HORIZONTALLY TO THE RIGHT IN THE
POSITIVE X DIRECTION

24 (b) The graph of $y = -3x^2 + 4x - 5$ is reflected in the y -axis. so $f(-x)$

Work out the equation of the reflected graph.

Give your answer in its simplest form.

[2 marks]

SUBSTITUTE $(-x)$ IN FOR x

$$\begin{aligned} y &= -3(-x)^2 + 4(-x) - 5 \\ &= -3x^2 - 4x - 5 \end{aligned}$$

Answer $y = -3x^2 - 4x - 5$

(or $y = -(3x^2 + 4x + 5)$)

END OF QUESTIONS

Turn over ►

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