

Please write clearly, in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

SOLUTIONS

# GCSE MATHEMATICS

# H

Higher Tier Paper 1 Non-Calculator

Exam Date

Morning

Time allowed: 1 hour 30 minutes

## Materials

For this paper you must have:

- mathematical instruments.

You must **not** use a calculator.



## Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

## Advice

- In all calculations, show clearly how you work out your answer.

Answer **all** questions in the spaces provided.

1

Expand

$$x(x+3) = x \times x + x \times 3 = x^2 + 3x$$

Circle your answer.

[1 mark]

$2x + 3$

$x^2 + 3$

$x^2 + 3x$

$3x^2$

2

Which of these has the greatest value?

Circle your answer.

[1 mark]

$6.15 \times 10^4$

61 499

$6.2 \times 10^3$

$61.6 \times 10^3$

$6.1500 \times 10^4$

$6.200 \times 10^3$

$61.6000 \times 10^3$

$= 61500$

61499

6200

$= 61600$

BIGGEST

- 3 What is 0.12 as a fraction of 0.8?  
Circle your answer.

$$0.12 \times 100 = 12$$

$$0.8 \times 100 = 80$$

so  $\frac{12}{80} \xrightarrow{\div 4} \frac{3}{20}$

[1 mark]

$$\frac{3}{20}$$

$$\frac{2}{3}$$

$$\frac{20}{3}$$

$$\frac{3}{2}$$

- 4 The base of a pyramid has  $n$  sides.  
Circle the expression for the number of faces of the pyramid.

[1 mark]

$$2n$$

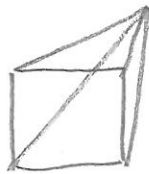
$$n-1$$

$$n$$

$$n+1$$

Turn over for the next question

eg 4 SIDES ON BASE, SO 5 SIDES ALTOGETHER

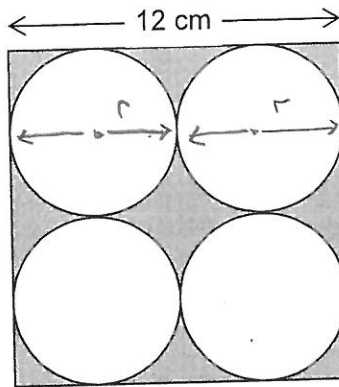


$$= 4 + 1$$

$$\text{so } n + 1$$

5

Four identical circles just fit inside a square as shown.

Not drawn  
accurately

$$r = 12 \div 4 = 3$$

Work out the area of the shaded section.

Give your answer in terms of  $\pi$ .

[4 marks]

$$\text{AREA OF SQUARE} = 12 \times 12 = 144$$

$$\text{AREA OF ONE CIRCLE} = \pi r^2 = \pi \times 3^2 = 9\pi$$

$$\text{AREA OF 4 CIRCLES} = 4 \times 9\pi = 36\pi$$

$$\text{SHADED AREA} = 144 - 36\pi$$

$$\text{Answer } \underline{144 - 36\pi} \text{ cm}^2$$

6

Bag A contains 10 blue balls and 20 red balls.

Bag B contains 8 blue balls and 12 red balls.



A ball is chosen at random from each bag.

Jo says,

"It is more likely that a blue ball is chosen from Bag A than Bag B because there are more blue balls in Bag A."

Is she correct?

You **must** show your working.

[3 marks]

$$\text{For (A): } P(\text{BLUE}) = \frac{10}{30} = \frac{1}{3} \quad \text{For (B): } P(\text{BLUE}) = \frac{8}{20} = \frac{2}{5}$$

SO COMPARE USING EQUIVALENT FRACTIONS, COMMON DENOMINATOR 15 (3x5)

$$\frac{1}{3} \xrightarrow{\times 5} \frac{5}{15} \quad \frac{2}{5} \xrightarrow{\times 3} \frac{6}{15}$$

ANSWER, NO, FOR BAG B, PROBABILITY IS HIGHER.

Turn over for the next question

7 (a) Rob is going to drive 130 miles from Hull to Liverpool.

There are road works for 25 miles of the journey.

He assumes his average speed will be

50 mph where there are road works

70 mph for the rest of the journey.



Using his assumptions, work out his journey time.

[4 marks]

TOTAL = 130 ① WITH ROADWORKS: 25 MILES  
 ② WITHOUT ROADWORKS: 130 - 25 = 105

~~DIS~~ TIME FOR ①  $T = \frac{D}{S} = \frac{25}{50} = 0.5$

TIME FOR ②  $T = \frac{D}{S} = \frac{105}{70} = \frac{21}{14} = \frac{3}{2} = 1.5$

TOTAL = 0.5 + 1.5 = 2 HOURS

Answer 2 HOURS or 120 MINUTES

7 (b) Rob's assumptions about his average speeds are too high.

How does this affect his journey time?

[1 mark]

THE JOURNEY WILL BE LONGER

8

Jack works out the answer to

$$\frac{\sqrt{98.5} - 12.1}{-0.8} \approx \frac{\sqrt{100} - 12}{-0.8} = \frac{10 - 12}{-0.8}$$

He says the answer is negative.

Is he correct?

$$\text{so } \sqrt{98.5} < 12$$

You **must** show your working.

[2 marks]

$$= \frac{10 - 12}{-0.8} = \frac{-2}{-0.8} \approx \frac{-2}{-1} = +2$$

NEGATIVE DIVIDED BY NEGATIVE = POSITIVE

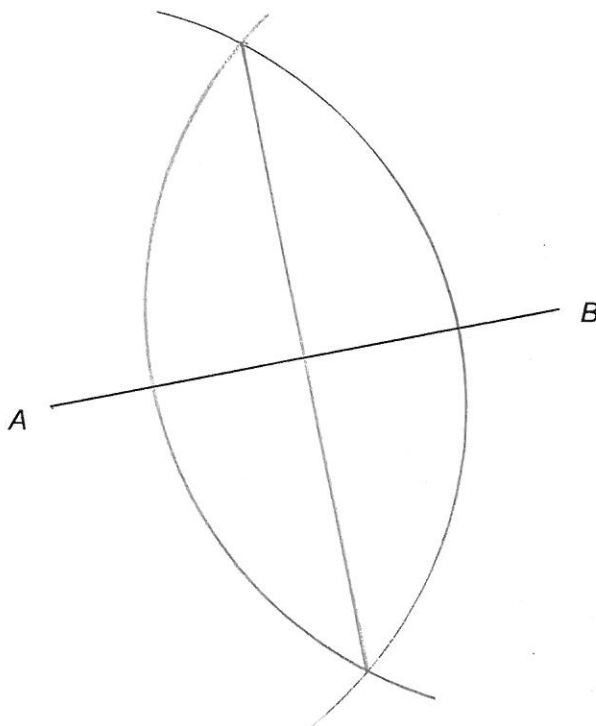
SO NOT CORRECT.

9

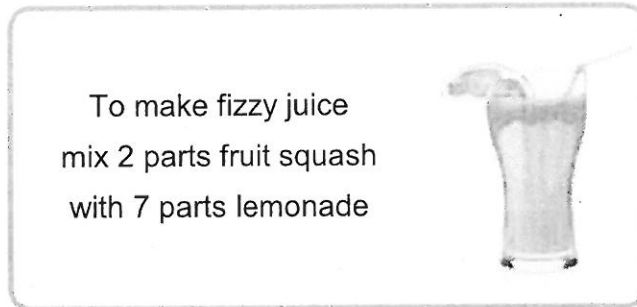
Use a ruler and a pair of compasses in this question.

Construct the perpendicular bisector of AB.

[2 marks]



- 10 Here are the instructions on a bottle of fruit squash.



S:L TOTAL PARTS  
2:7 9

Tom has 80 ml of fruit squash.

He also has 210 ml of lemonade.

What is the **maximum** amount of fizzy juice he can make?

[3 marks]

① 
$$\begin{array}{l} S:L \\ 2:7 \\ \times 40 \quad \left\{ \begin{array}{l} 80:280 \end{array} \right. \times 40 \\ \hline \end{array}$$
 X

② 
$$\begin{array}{l} S:L \\ 2:7 \\ \times 30 \quad \left\{ \begin{array}{l} 60:210 \end{array} \right. \times 30 \\ \hline \end{array}$$
 ✓

① CANT WORK, ONLY 210ml OF LEMONADE

SO MAXIMUM FROM ② IS 60ml + 210 ml

Answer 270 ml ml

- 11 A ball is dropped from a height of 50 metres.  
After each bounce, the ball reaches 20% of its previous height.

How high does it reach after the second bounce?

[2 marks]

$10\% \text{ of } 50 = 50 \div 10 = 5$

$20\% \text{ of } 50 = 2 \times 10\% = 2 \times 5 = 10$

~~50 - 10 = 40~~ 10m = HEIGHT AFTER FIRST BOUNCE

$20\% \text{ of } 10 = 10 \div 10 \times 2 = 2\text{m}$

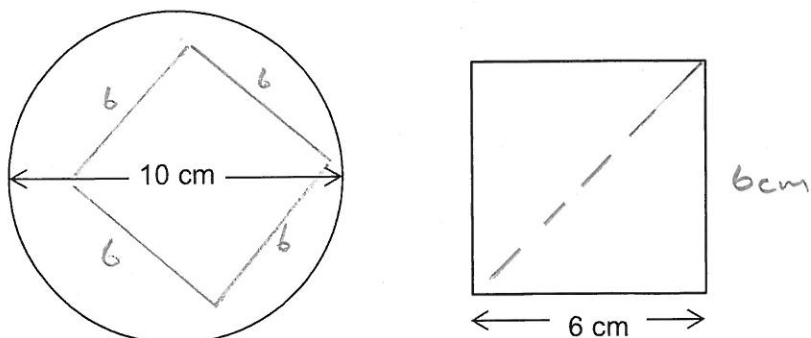
So 2m

Answer 2 metres



- 12 A circle has diameter 10 cm  
A square has side length 6 cm

Not drawn accurately



Use Pythagoras' theorem to show that the square will fit inside the circle without touching the edge of the circle.

[3 marks]

$$\begin{aligned} &\text{LENGTH OF DIAGONAL OF SQUARE} \\ &= \sqrt{6^2 + 6^2} = \sqrt{36 + 36} = \sqrt{72} \\ &\sqrt{72} < 10 \text{ cm} \end{aligned}$$

AS DIAGONAL IS SHORTER THAN DIAMETER, SQUARE SHOULD FIT IN AS SHOWN ABOVE

- 13 What percentage of a distribution is covered by the inter-quartile range?  
Circle your answer.

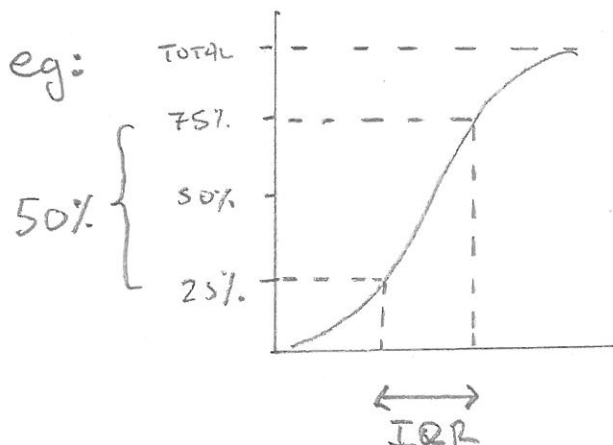
[1 mark]

25%

37.5%

50%

75%



- 14 Which of these values **cannot** be the cosine of an angle?  
Circle your answer.

[1 mark]

-0.5

0

0.5

1.5

- 15 A motor racing circuit has length  $5\frac{5}{6}$  miles.

A straight section of the circuit has length  $1\frac{3}{4}$  miles.

What fraction of the circuit is the straight section?

Give your answer in its simplest form.

[3 marks]

$$\text{TOTAL } 5\frac{5}{6} = \frac{35}{6} \quad \text{STRAIGHT} = \frac{1\frac{3}{4}}{= \frac{7}{4}}$$

$$\text{FRACTION} = \frac{7/4}{35/6}$$

$$= \frac{7}{4} \div \frac{35}{6} = \frac{7}{4} \times \frac{6}{35} \quad \text{CANCEL DOWN}$$

Answer  $\frac{3}{10}$

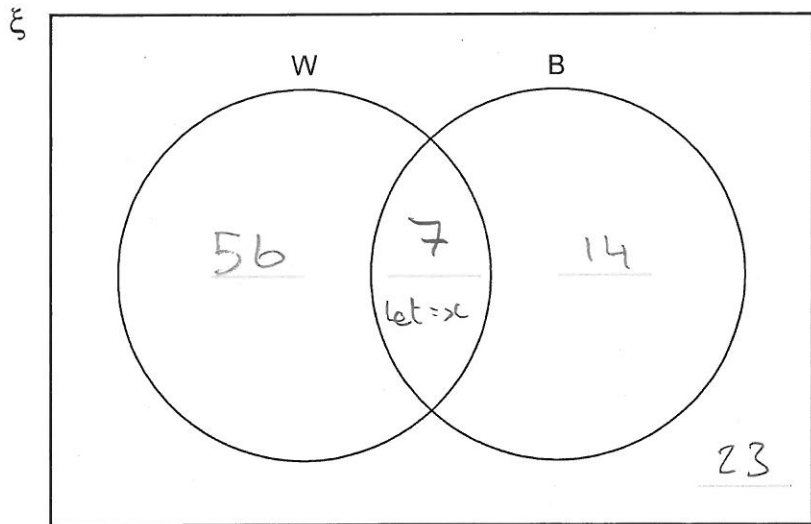
$$\begin{array}{l} 1 \\ 2 \end{array} \frac{7 \times 6}{4 \times 35} = \frac{1 \times 3}{2 \times 5} = \frac{3}{10}$$

16 In the Venn diagram

$\xi = 100$  farms

W = farms that grow wheat

B = farms that grow barley



$$70 - 56 = 14$$

70 farms grow **only** wheat or **only** barley.

$\frac{4}{5}$  of these 70 farms grow **only** wheat.

$$\frac{4}{5} \times 70 = \frac{4 \times 70}{5} = 4 \times 14 = 56$$

The number of farms that grow wheat is three times the number that grow barley.

$$\begin{array}{r} 14 \\ \times 3 \\ \hline 42 \end{array}$$

Complete the Venn diagram.

3 W FOR 1 BARLEY = FOUR PARTS

[5 marks]

$$56 + x = 3(14 + x)$$

$$\begin{array}{r} 56 + x = 42 + 3x \\ -42 \quad -x \quad -42 \quad -x \end{array}$$

$$14 = 2x$$

Turn over for the next question

$$7 = x$$

$$\begin{array}{r} 56 \\ + 14 \\ + 7 \\ \hline 77 \end{array}$$

$$\begin{array}{r} 100 \\ - 77 \\ \hline 23 \end{array}$$

17

$$(3x+1)(x-2) + ax + b \equiv 3x^2 + 8x - 5$$

Work out the values of  $a$  and  $b$ .

[4 marks]

$$3x^2 - 6x + x - 2 + ax + b \equiv 3x^2 + 8x - 5$$

$$3x^2 - 5x - 2 + ax + b$$

$$3x^2 - 5x + ax + b - 2 \equiv 3x^2 + 8x - 5$$

$$\therefore 3x^2 \equiv 3x^2$$

$$-5x + ax \equiv +8x \qquad b - 2 \equiv -5$$

$$\text{so } x(a-5) = 8x \qquad b = -3$$

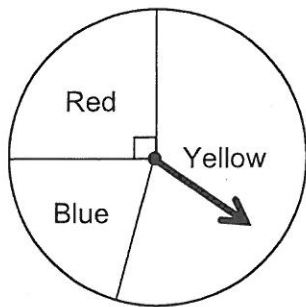
$$a - 5 = 8$$

$$a = 13$$

$$a = 13$$

$$b = -3$$

18 In a game, a fair spinner has three sections.



Not drawn accurately

18 (a) Joe uses this method to work out the probability of getting two reds from two spins.

He writes,

*There are three colours, so the probability of the spinner landing on red is  $\frac{1}{3}$*

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}, \text{ so the probability is } \frac{2}{3}$$

Make **two** criticisms of Joe's method.

[2 marks]

Criticism 1  $P(\text{RED}) = \frac{1}{4}$ , NOT  $\frac{1}{3}$

Criticism 2 THE PROBABILITIES SHOULD BE MULTIPLIED TOGETHER, NOT ADDED

18 (b) The probability of getting two blues from two spins is  $\frac{1}{25}$

Work out the angle of the blue sector.

[3 marks]

$P(B) = \frac{1}{25} = \frac{1}{5} \times \frac{1}{5}$  SO BLUE =  $\frac{1}{5}$  OF THE SPINNER.

So,  $\frac{1}{5} \times 360 = 72^\circ$   $\begin{array}{r} 72 \\ 5 \overline{) 360} \end{array}$

Answer 72° degrees

- 19 Show that  $\frac{2x+1}{3} + \frac{5x-2}{2}$  simplifies to  $\frac{19x-4}{6}$

[2 marks]

$$\frac{2(2x+1) + 3(5x-2)}{6}$$

$$= \frac{4x+2+15x-6}{6}$$

$$= \frac{19x-4}{6}$$

- 20 A circle has equation  $x^2 + y^2 = \frac{1}{4}$

Circle the length of its radius.

[1 mark]

$\frac{1}{16}$

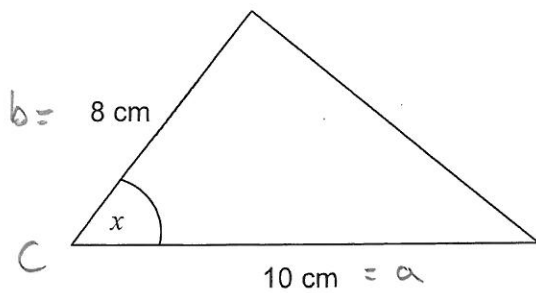
$\frac{1}{8}$

$\frac{1}{4}$

$\frac{1}{2}$

$$r = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

- 21 Which expression gives the area, in  $\text{cm}^2$ , of this triangle?



Not drawn accurately

$$A = \frac{1}{2} ab \sin c$$

$$A = \frac{1}{2} \times 10 \times 8 \times \sin x$$

$$A = 40 \sin x$$

Circle your answer.

[1 mark]

$80 \sin x$

$40 \sin x$

$80 \cos x$

$40 \cos x$

- 22 Express  $0.\dot{1}5$  as a fraction in its simplest form.

[3 marks]

$$\text{let } x = 0.1555 \dots$$

$$10x = 1.555 \dots$$

$$100x = 15.555 \dots$$

$$-(10x = -1.555 \dots)$$

$$90x = 14$$

$$x = \frac{14}{90} = \frac{7}{45}$$

Answer

$$\frac{7}{45}$$

23

Ellie bought a scarf and a dress.

The scarf cost £4

She sold both items for a total of £26

She made

100% profit on the cost of the scarf

30% profit on the total cost.

Work out her percentage profit on the cost of the dress.

[5 marks]

$$\text{SCARF: COST} = \underline{\pounds 4} \quad 100\% \text{ PROFIT} = \underline{\pounds 4}$$

$$\text{SCARF SOLD FOR } \underline{\pounds 4 + \pounds 4 = \pounds 8}$$

30% PROFIT ON TOTAL COST:

$$\text{TOTAL COST} \times 130\% = \underline{\pounds 26} \quad \text{OR } \text{TOTAL} \times 1.3 = \underline{\pounds 26}$$

$$\text{SO } \underline{\pounds 26} \div 1.3 \quad [= \quad 260 \div 13 = \underline{\pounds 20}]$$

$$\text{SO ORIGINAL COST} = \underline{\pounds 20}$$

$$\text{TOTAL PROFIT} = \underline{\pounds 26} - \underline{\pounds 20} = \underline{\pounds 6}, \quad \underline{\pounds 6} - \underline{\pounds 4} \text{ SCARF} = \underline{\pounds 2}$$

$$\text{PERCENTAGE PROFIT} = \left( \frac{\text{ACTUAL PROFIT}}{\text{ORIGINAL COST}} \right) \times 100$$

$$\underline{\pounds 26} - \underline{\pounds 8} = \underline{\pounds 18} \quad \underline{\pounds 18} - \underline{\pounds 2} = \underline{\pounds 16} = \text{COST OF DRESS}$$

Answer 12.5 %

$$\% \text{ PROFIT} = \frac{\underline{\pounds 2}}{\underline{\pounds 16}} \times 100 = \frac{1}{8} \times 100 = 12.5$$



24

Work out  $\sqrt[3]{8} \times 5^{-2}$ 

Give your answer as a decimal.

[3 marks]

$$\sqrt[3]{8} = 2$$

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$2 \times \frac{1}{25} = \frac{2}{25} = \frac{8}{100} = 0.08$$

Answer 0.08

Turn over for the next question

25 A sequence of numbers is formed by the iterative process  $a_{n+1} = (a_n)^2 - a_n$

25 (a) Describe the sequence of numbers when  $a_1 = 1$

Show working to justify your answer.

$$a_2 = (1)^2 - 1 = 0$$

$$a_3 = 0$$

$$a_4 = 0$$

!

[1 mark]

AFTER  $a_2 = 0$ , ALL THE NUMBERS ARE 0

25 (b) Describe the sequence of numbers when  $a_1 = -1$

Show working to justify your answer.

$$a_2 = (-1)^2 - (-1) = 1 + 1 = 2$$

$$a_3 = (2)^2 - 2 = 2$$

$$a_4 = (2)^2 - 2 = 2$$

[2 marks]

AFTER  $a_1 = -1$ , ALL VALUES ARE 2

25 (c) Work out the value of  $a_2$  when  $a_1 = 1 - \sqrt{2}$

$$a_2 = (1 - \sqrt{2})^2 - (1 - \sqrt{2})$$

[2 marks]

$$(1 - \sqrt{2})^2 = (1 - \sqrt{2})(1 - \sqrt{2})$$

$$= 1 - \sqrt{2} - \sqrt{2} + 2$$

$$a_2 = 3 - 2\sqrt{2} - 1 + \sqrt{2}$$

$$= 1 - 2\sqrt{2} + 2$$

$$= 2 - \sqrt{2}$$

$$= 3 - 2\sqrt{2}$$

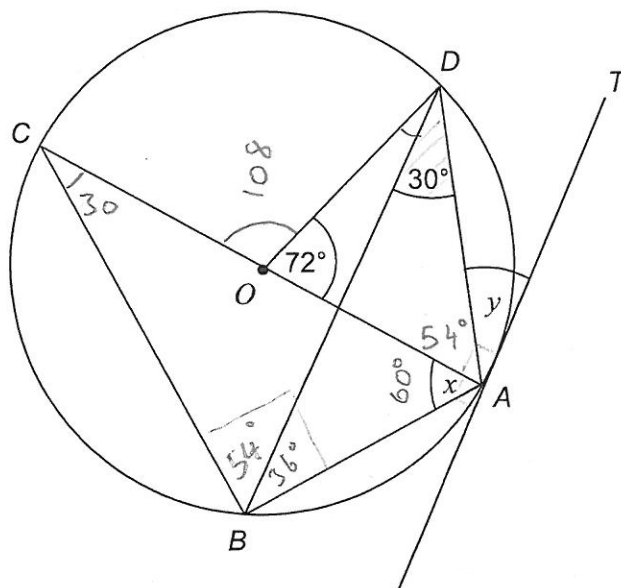
Answer 2 - \sqrt{2}

- 26  $A, B, C$  and  $D$  are points on a circle, centre  $O$ .  
 $AC$  is a diameter of the circle.  
 $AT$  is a tangent to the circle.

Not drawn  
accurately

$$\angle C = \angle D$$

$$30 = 30$$



Work out the size of angle  $x$  and the size of angle  $y$ .

[4 marks]

$$\angle C = 30 \quad \angle A = \angle BDA$$

$$x = 180 - 90 - 30 = 60^\circ$$

$$\angle BDA = 36 \quad \text{so } \angle BAD = 180 - 30 - 36 = 114$$

$$\angle DAC = 114 - 60 = 54^\circ$$

$$y = 90 - 54 = 36^\circ$$

$$x = \underline{60^\circ} \text{ degrees}$$

$$y = \underline{36^\circ} \text{ degrees}$$

27 Write  $\sqrt{12} + \frac{15}{\sqrt{3}}$  in the form  $a\sqrt{b}$  where  $a$  and  $b$  are prime numbers.

[3 marks]

RATIONALISE  $\frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{3}$

$$\begin{aligned} \sqrt{4 \times 3} &= \sqrt{12} = 2\sqrt{3} & \text{so } 2\sqrt{3} + \frac{15\sqrt{3}}{3} &= 2\sqrt{3} + 5\sqrt{3} \\ &= \sqrt{4 \times 3} & & \\ &= 2\sqrt{3} & & = 7\sqrt{3} \end{aligned}$$

$$a = 7 \quad b = 3$$

Answer  $7\sqrt{3}$

28 Solve  $5x - y = 5$  REARRANGE BOTH EQUATIONS TO  $2y = \dots$   
 $2y - x^2 = 11$

You **must** show your working.

Do **not** use trial and improvement.

[6 marks]

$$\begin{array}{r} 5x - y = 5 \\ \hline \begin{array}{r} +y \\ 5x = y + 5 \\ \hline -5 \quad -5 \end{array} \end{array}$$

$$\begin{array}{r} 2y - x^2 = 11 \\ \hline \begin{array}{r} +x^2 \quad +x^2 \end{array} \end{array}$$

$$2y = x^2 + 11$$

$\times 2$   $(5x - 5 = y)$

$$10x - 10 = 2y$$

Let  $2y = 2y$  so  $10x - 10 = x^2 + 11$

or

$$\begin{array}{r} x^2 + 11 = 10x - 10 \\ \hline \begin{array}{r} -10x + 10 \quad -10x + 10 \end{array} \end{array}$$

$$x^2 - 10x + 21 = 0$$

FACTORISE  $(x - 3)(x - 7) = 0$

$$x = 3, x = 7$$

SUBSTITUTE INTO  $5x - y = 5$  OR  $5x - 5 = y$  FROM ABOVE

$$\begin{array}{r} x = 3: 5 \times 3 - 5 = y \\ \hline 15 - 5 = y \\ \hline 10 = y \end{array}$$

$$\begin{array}{r} x = 7: 5 \times 7 - 5 = y \\ \hline 35 - 5 = y \\ \hline 30 = y \end{array}$$

so  $x = 3, y = 10$        $x = 7, y = 30$

Answer \_\_\_\_\_

CHECK IN 2nd EQN  $2y = x^2 + 11$

END OF QUESTIONS

$$\begin{array}{r} x = 3 \\ 2y = 3^2 + 11 \\ 2y = 20 \\ y = 10 \checkmark \end{array}$$

$$\begin{array}{r} x = 7 \\ 2y = 7^2 + 11 \\ 2y = 60 \\ y = 30 \checkmark \end{array}$$